The Correlation between Practice Time and Student Improvement in Mathematics

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Structured Abstract

Context

As all engineers are fully aware the mathematics in engineering courses is used not only for calculating solutions to problems, but for many other functions. Mathematics is a language for understanding, a language for teaching, and most importantly it is a language that makes self-study and continuing professional development easier. It is therefore necessary that engineers know not only various mathematical procedures but also understand them and are fluid in them, i.e. can use them easily and almost without thinking. In order to get this fluidity, it is necessary that students are exposed to two processes when learning mathematics:

- The student must be given lectures which explain the important concepts in the relevant mathematics clearly. Further, these lectures must be given by a lecturer that understands the structure of the particular mathematical topic being covered and who understands the links between the topics being covered and the other mathematical topics that engineers need to know.
- The students need to undertake *directed* practice with relevant *feedback* in the mathematics that they need to know.

Purpose

The hypothesis of this paper is that in order to become fluid in mathematics the student needs to spend time working on tutorial problems.

Approach

This paper will present data showing that the time spent on directed practice of tutorial problems is highly correlated with the improvement in the students' marks. Using a proprietary computer package (MyMathLab Global by Pearson Publishers) to obtain practice time and the difference between the marks of a diagnostic mathematics test in the first week of the semester and the marks in the final exam a correlation analysis will be undertaken.

Results

This analysis will show that the more time that a student spends on directed practice the greater will be their improvement in marks.

Conclusions

A problem with this study is that correlation is not causation and other factors may be influencing the correlation. The paper will discuss these points in detail and show that the correlation analysis is likely to have a high validity and that the initial hypothesis is reasonable.

Keywords: Directed-practice, feedback, practice-time.

Introduction

As all engineers are fully aware, the mathematics in engineering courses is used not only for calculating solutions to problems, but for many other functions. Mathematics is a tool for seeing patterns and interconnections, it is a tool for facilitating understanding, a language for teaching and explaining technical concepts, and most importantly it is a language that facilitates self-study and continuing professional development. Modern technology is changing at such a rate that many products and processes are obsolete within a short time. It is therefore essential that engineers have the tools to enable them to keep up to date: mathematics is one of the most important of these tools.

It is therefore necessary that engineers know various mathematical procedures but, more importantly, that they understand them and are fluid in them, i.e. can use them easily and almost without thinking.

Background to the Study

The Importance of Working Memory for Learning

When students are learning a new concept, it is their working memory that is being used to understand the new topic and relate it to other relevant concepts (Baddeley, 2004). Unfortunately, human working memory is limited. It can hold about 7 independent items in storage at a time (Baddeley, 2004). Therefore, when teaching a new topic, lecturers must take care not to fill the students' working memory with items that are not directly related to the concept being taught (Barclay, Bransford, Franks, McCarrel, and Nitsch 1974). If a lecturer is using mathematics as an aid to explain a new engineering concept, the mathematics that the lecturer uses must be mathematics that the students are fluid with. That is, mathematics that the students have stored in their long-term memories because long term memories have the property that they are able to access concepts rapidly, subconsciously and almost without thinking (Willingham, 2009). This is necessary so that the lecturer can explain the new concept without the student having to use up limited working memory to understand the mathematics underlying the new concept before they can direct their working memory to the new concept. If the students are using their limited working memory to understand the underlying mathematics they will not have sufficient reserves of working memory to allocate to fully understanding the new engineering concept which, in turn, will be detrimental to their learning (Cumming and Elkins, 1999).

In order to make the mathematics fluid and to prevent it from using up limited working memory while teaching other engineering concepts it is necessary that the students have transferred the key mathematical concepts to their long-term memory. This is achieved via directed practice of the basic mathematical procedures and concepts together with regular feedback (Willingham, 2009).

Becoming an Expert

The research into the abilities and creation of experts is relevant to the above discussion. Ericson, et.al. have shown that in order to become an expert in a particular area such as violin playing, chess, etc. it is necessary for the average person to spend about 10 000 hours of directed, goal oriented practice with regular feedback (see graph below) (Ericson, Kampe, and Tesch-Romer, 1993). In addition, a person with a background of directed practice in an area has increased ability to concentrate on topics in that area (Brown, Roediger III, McDaniel, 2014). Anecdotally, many of our students seem to have difficulty with engagement and concentration.

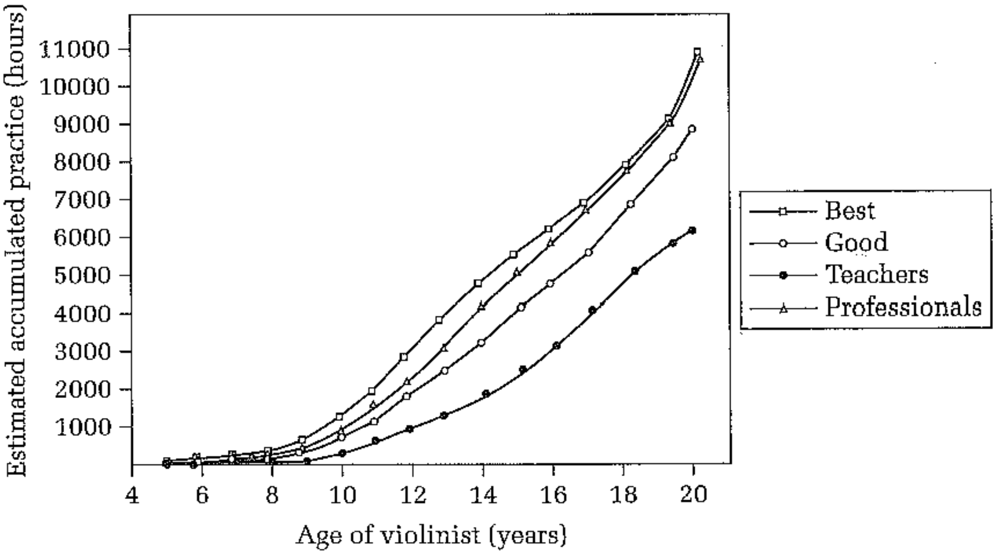

Figure 1: Ericson's data for violinists' practice time (Ericson, et.al., 1993)

An expert in a particular field has the ability to see the solution to a problem in that field rapidly and unconsciously without using significant amounts of working memory (Ericson, Kampe, and Tesch-Romer, 1993). Lecturers are experts in their fields and therefore need to keep the above in mind when dealing with students learning a new topic. That is, what is automatic and immediately obvious to the lecturer is unlikely to be so to the student (Willingham, 2009).

Aims of Directed Practice

As discussed above, the overall aims of directed practice in mathematics is to make the students fluid and automatic with mathematics, and to have the ability to retrieve basic mathematical concepts rapidly and subconsciously, (Alexander, Kulikowich, and Schulze, 1994). In addition, it has been found that students that are fluid with mathematics are more likely to see the deep structure within mathematics than those who do not have mathematical fluidity (Schacter, 2002).

In order for practice to be effective it must have the following characteristics:

- The person practicing must practice beyond perfection (Bahrick and Hall, 1991). That is, the person practicing must not stop practicing when she can do a particular practice item once correctly but repeat it a number of times.
- The practice must be directed, have a goal and receive regular feedback (Kang, McDermott, and Roediger, 2007; Gladwell, 2008).
- The basic concepts underlying the practice item must be regularly reviewed (Ellis, Semb, and Cole, 1998; Bahrick, and Hall, 1991).
- The practice should be distributed in time and not concentrated in one long session (Soderstrom and Bjork, 2014).
- The learners must concentrate on what they are doing and think about what they are doing (Willingham, 2009).
- Practice of different concepts should be interleaved with each other rather than doing one concept then the next, etc. (Brown, et.al, 2014).

Study Conducted at the Manukau Institute of Technology

Structure of the Study

The study that is the subject of this paper was conducted at the Manukau Institute of Technology, it involved 31 students, and it dealt with the first-year mathematics paper: Engineering Mathematics 1 (141.514). This paper is the first of two one-semester mathematics papers studied during the three-year engineering degree at the polytech. A one semester paper runs for 15 weeks and consists of 3 hours of lectures plus one tutorial hour per week. In addition, the students are expected to do 6 hours of self-study per week per subject.

During week one the students were given a diagnostic test based on the year-eleven school mathematics syllabus. The reason for choosing the year-eleven material was the students have to have year-thirteen mathematics to get entry to the three-year engineering degree and, therefore, should be able to do year-eleven school mathematics without any difficulty. The mark obtained by the students on the diagnostic test was then compared with their mark in the final end-of-semester exam as described below. It is interesting to note that 16 out of 31 students failed (< 50%) this diagnostic test and the average mark was only 45.6%. This confirms anecdotal evidence claiming the schools are not adequately preparing students for tertiary study.

In order for the students to get directed practice in the basic mathematic concepts an online package published by Pearson's (MyMathLab Global) was used. A test bank of 11 quizzes with each quiz consisting of about 30 questions was set up. Each quiz covered a major topic in the Mathematics 1 syllabus, e.g. matrices. The quizzes were allocated 15% of the students' final mark to encourage the students to do the quizzes, i.e. seven quizzes were allocated 1% each and 4 quizzes, on more important topics, were allocated 2% each. In addition to the quizzes the students sat three class tests worth 35% in total and an end-of-semester exam worth 50%.

The quizzes were done in a collaborative environment in order to get regular feedback. That is, the students could discuss the problems with each other, they had access to a tutor for one hour per week, and the online package had help functions. In addition, the students could do the quizzes off campus and could get help from family, friends, etc.

The online help functions for the quizzes consisted of access to an e-book that automatically provided the students with a textual explanation of the theory behind the problem that they were currently working on. In addition, the online help had a "hint" function that showed the students a step-by-step procedure for any problem that they were currently working on.

The students could do the quizzes as many times as they wished and the highest mark achieved for any particular quiz was recorded as the student's mark. Each quiz question for a particular topic had the same mathematical structure but different numerical values so each student did a numerically different set of quiz questions for each topic.

As discussed above, the aim of this directed practice was to improve the fluidity of the students with basic mathematics concepts.

Results of the Study

Detailed results comparing the improvement in the student marks with the time spent on the quizzes are given in the appendix below.

As shown in a previous paper (Shepstone, 2016), the effect size of the improvement in student marks from the diagnostic test to the final exam was 0.7 (0.4 is regarded as good in the educational setting, Hattie (2009)) and the Student's t-test showed that the means of the diagnostic test marks and the final exam marks were significantly different. These results showed that the students had significantly improved their mathematical performance between the diagnostic test and the end-of-semester exam.

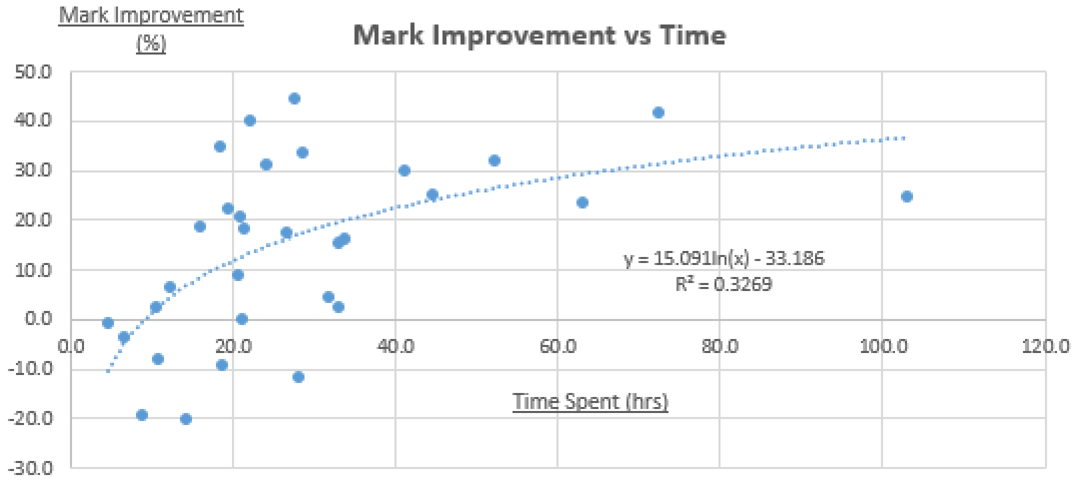

Figure 2: Mark improvement vs time regression analysis

The current study compared the improvement in the student's results between the diagnostic test and the end-of-semester exam with the amount of time spent by the students on the quizzes as shown in the graph above.

To do the analysis shown in the graph it was assumed that the students' improvement in mathematics marks relative to the time spent on the quizzes would follow a traditional learning curve. That is, the learning would be more rapid initially and then increase at a diminishing rate as more time was spent on the quizzes. Therefore, a log curve was fitted to the data as shown on the graph; the equation of this curve is also shown on the graph. The R^2 of this curve is 0.33 showing that this curve explains 33% of the variation in the data. In other words, it may be hypothesised that the time spent on the eleven quizzes explained 33% of the improvement in the students' marks. Alternatively, Figure 2 shows that for an average student to improve her marks by, for example, 22% she needs to spend 40 hours working on the quizzes.

Discussion of the Results and Limitations of the Study

Considering that the quizzes made up only 15% of the students' final mark this result shows that the quizzes had a proportionally large effect on the students' results.

The remaining 66% of the variation in the data was probably due to a number of factors. As the graph shows, a number of students made improvements in their marks that were significantly better than the regression curve. This could be because those students spent time exploring why they got a question wrong or they engaged with the lectures more effectively and thus made more rapid progress. Also, some students have not studied mathematics for a number of years and the quizzes may have provided a reminder and revision of material they already knew. That is, this material may have already been stored in their long-term memory and the quizzes brought it to the surface again.

Other students made improvements in their marks that were significantly worse than the regression curve and, in a few cases, were even negative. This negative variation in the data could be because some students found the volume of work involved in the Engineering Mathematics 1 course was too much and they became more confused as the course progressed. In addition, these students may not have engaged with the lectures and may not have tried to understand why they had got problems wrong on the quizzes but merely continued to the next problem. The time spent on a quiz is a proxy measure for how well the students actually engaged with the quiz. It therefore, does not indicate how well this time was utilised for effective learning. As the graph below shows (the horizontal axis is the student number) some students' marks improved in direct relation to the time that they spent on the quizzes whereas other students' marks showed a negative correlation indicating that these students did not engage effectively with the quizzes.

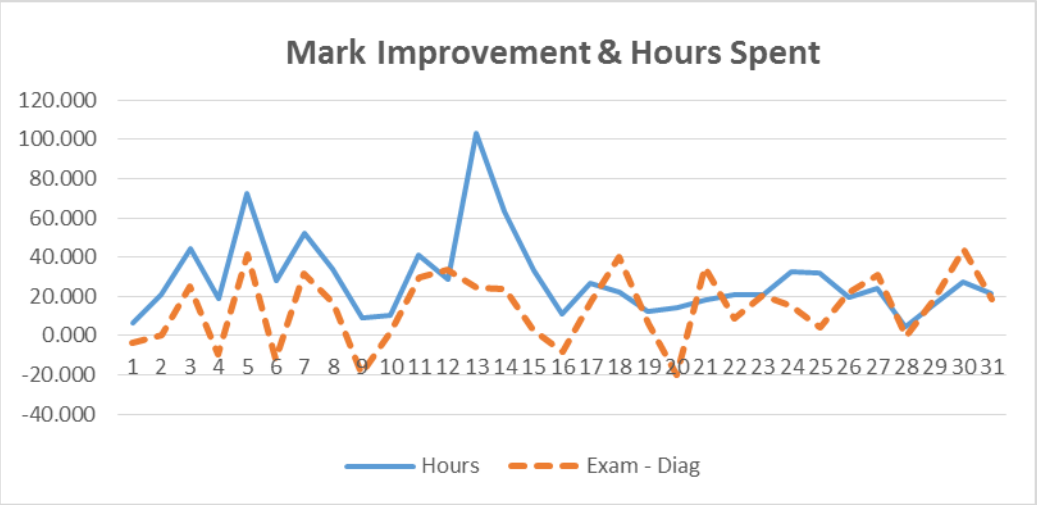

Figure 3: Mark improvement and time spent on quizzes.

In addition, the data in the appendix shows that the average student spent 1 hour 7 minutes per week working on the quizzes in excess of the one hour timetabled tutorial time. Considering that students are expected to spend 6 hours per week outside of timetabled time working on each subject this 1 hour and 7 minutes is not impressive. (Admittedly, the students also had to study for the three tests and the final examination but it is unlikely that they spent 4 hours and 53 minutes on these activities per week.)

This study has a number of limitations. Firstly, correlation is not causation, however it is reasonable to hypothesise that time spent doing quizzes has a casual effect on the students' results. Secondly, this study is small as it involves only 31 students so it shall be run for a number of semesters to see whether these results are robust. In addition, it would be advantageous for an independent polytech to repeat the study to see if they obtain similar results. Thirdly, this study was not a blinded study because of ethical considerations. The ethics committee required that all the students be taught in the most effective way possible which meant that all the students had to do the quizzes and it was not acceptable to divide the study into two halves with only half the group doing the quizzes. Fourthly, the end-of-semester examination was significantly more difficult than the diagnostic test because it included topics such as calculus, matrices, and complex numbers which the diagnostic test did not. Therefore, these results are probably an underestimate of the improvement that the students made. Finally, this study ran for only 15 weeks which is a short time for any substantial improvement in a student's mathematical ability to be made, particularly in the light of Ericson's work which indicates that substantial amounts of time on directed practice is needed to make major improvements in one's abilities.

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Appendix

Table 1: Raw data used for analysis

	BET Maths Results (2016-1, 2016-2, 2017-1)						
		Time Spent					
Student No.	H:Min:Sec	Days	Hours	**Exam mark**	**Diagnostic Mk**	**Estimated**	**Exam - Diag**
7910	6:32:47	0.27277	6.546	38.8	42.3	50.0	-3.5
2274	21:08:16	0.88074	21.138	34.7	34.6	75.0	0.1
9222	44:41:30	1.86215	44.692	72.2	47.4	60.0	24.8
3598	18:39:53	0.77770	18.665	70.0	79.5	90.0	-9.5
2079	72:28:47	3.01999	72.480	64.7	23.1	40.0	41.6
9447	28:02:41	1.16853	28.045	53.5	65.4	30.0	-11.9
9613	52:18:31	2.17953	52.309	44.6	12.8	10.0	31.8
6627	33:52:04	1.41116	33.868	20.2	3.9	10.0	16.3
7066	8:49:21	0.36760	8.823	42.3	61.5	60.0	-19.2
9450	10:42:14	0.44600	10.704	17.9	15.4	10.0	2.5
0545	41:04:28	1.71144	41.074	46.7	16.7	50.0	30.0
6575	28:33:47	1.19013	28.563	96.3	62.8	75.0	33.5
5358	103:12:53	4.30061	103.215	47.6	23.1	50.0	24.5
3406	63:02:56	2.62704	63.049	85.0	61.5	50.0	23.5
4856	33:06:53	1.37978	33.115	66.7	64.1	87.0	2.6
0466	10:51:01	0.45209	10.850	65.0	73.0	70.0	-8.0
2063	26:39:50	1.11100	26.664	73.4	56.0	70.0	17.4
9450	22:09:39	0.92337	22.161	75.0	35.0	10.0	40.0
0313	12:18:36	0.51292	12.310	46.4	40.0	80.0	6.4
1742	14:24:26	0.60030	14.407	55.8	76.0	60.0	-20.2
2016	18:28:37	0.76987	18.477	69.7	35.0	75.0	34.7
6210	20:39:43	0.86091	20.662	62.9	54.0	50.0	8.9
9467	20:50:16	0.86824	20.838	66.7	46.0	40.0	20.7
6326	32:55:29	1.37186	32.925	73.4	58.0	60.0	15.4
0063	31:46:34	1.32400	31.776	87.5	83.0	85.0	4.5
1535	19:22:57	0.80760	19.383	82.1	60.0	60.0	22.1
2991	24:13:32	1.00940	24.226	45.0	14.0	30.0	31.0
8554	4:35:33	0.19135	4.593	3.3	4.0	25.0	-0.7
5517	16:01:36	0.66778	16.027	87.5	69.0	85.0	18.5
0088	27:29:42	1.14563	27.495	71.3	27.0	50.0	44.3
1865	21:21:45	0.89010	21.363	86.3	68.0	60.0	18.3
		Average	28.724	59.8	45.6	53.5	14.2
		Home av/week	1.115				
		Std dev	20.8	22.2	23.4	24.0	17.9
	T-test shows means are significantly different between the exam mark and the diagnostic mark.						