An Analysis of the Teaching of Mathematics in Undergraduate Engineering Courses.

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***Abstract**: The main objective of this paper is to discuss methods that can be used to inspire engineering students to learn engineering mathematics. Many of these issues relate to the rapid advancement in computer technology and the development of a branch of mathematics which involves generating solutions to complex mathematical equations using computers (computational mathematics). Arguably, this branch of mathematics is becoming more important to engineers than conventional (traditional) analytical mathematics. Another issue is the emergence of Computer Algebra Software (CAS) packages, which are software packages that can perform elaborate and complicated analytical mathematics much faster than a human ever can. The role of CAS in engineering education is discussed. Another important matter that is covered here is the issue of how maths should be incorporated into the engineering curriculum. The concept of how mathematics should be taught ``in context" as part of an engineering science subject will be explored.*

Introduction

Engineering is a very practical, hands-on occupation. Society has a vision of practicing engineers collaborating with other professions and working in teams so that they can ``do stuff" and make things work better. Thus, many senior high school students (rightfully!) envisage that hands-on practical experience with tangible engineering systems and working in teams are a big part undergraduate engineering experience at prestigious universities around the world. However, in reality, engineering courses at most elite institutions require students to learn a large amount of theory that involves the teaching and learning of complex and advanced mathematics with very little practical component. The traditional and conventional teaching of engineering mathematics encourages learning by demanding students sit in isolation and learn by repetition, echoing and memorizing the steps taken by their lecturers to solve very theoretical problems in applied mathematics. This model directly contradicts the student's (and society's) vision of the activities involved in engineering education. To make matters worse, mathematics is usually taught ``out of context", without any reference to the underlying practical engineering problems. As a result, many undergraduate students lose motivation, which leads to a lack of understanding of fundamental mathematics in engineering. Without a strong math background, students usually have a great level of difficulty in other engineering subjects ultimately resulting in a very low level of satisfaction with the overall engineering program.

In general, all engineering students should be familiar with the following maths topics, *Single and multi variable calculus*, *Vector Calculus*, *Linear Algebra*, *Ordinary Differential Equations*, *Partial Differential Equtions* and basic *Statistics*. Usually, all these topics are taught in separate mathematics subjects. They are usually not incorporated into the teaching of engineering subjects. When maths lecturers/professors introduce these topics in lectures, he/she usually does not make any reference to the underlying engineering physics or practical problem that lead to the mathematical equations. Students thus find it difficult to recognise the relevance of learning all these difficult methods to solve a set of equations. One of the reasons for this problem is because mathematics is usually taught as an isolated subject, distancing itself from the engineering science subjects that generate mathematical equations and uses the solution to the equations. The main reason for teaching maths as a separate subject is that many engineering disciplines need similar mathematics for solving similar sets of equations. This model is efficient from an administrative perspective, but the pedagogical benefits to the students are questionable.

There are more issues associated with the teaching of mathematics than merely deciding on how the teaching of mathematics can ``co-exist" with other engineering subjects. There are also issues associated with what kind of mathematics should be taught. In a general sense, engineering mathematics can be broadly divided into two different streams, *analytical* (``pen and paper") and *computational/numerical* mathematics. *Analytical* mathematics is the mathematics that we have grown accustomed to in primary and high school where a mathematical equation is solve using ``pen and paper". Mathematical problems that can be solve using *analytical* mathematics is usually relatively simple. On the other hand, *computational* mathematics is an approximation technique that leads to algorithms that must be implemented on a computer to solve complex mathematical equations. In recent years, with the advancement of computer technology, there are additional pressures for engineers to be proficient in using commercial engineering software to solve very complex problems. These complex problems are usually solved utilising computer algorithms developed using *computational* mathematics. Thus, it is important for engineers to understand *computational* mathematics and learn the origin of the algorithms used in many commercial engineering software. The main drawback of *computational* mathematics is that it is an approximation technique, which generates an **estimate** of the exact solution to the math problem. The pure mathematician would argue that the solution is not rigorous (because it is not exact) and does not give insight into the underlying physics but many practicing engineers would argue that the solution from *computational* mathematics is ``good enough" for real-life applications.

Another important issue to consider in the teaching of mathematics is the emergence of Computer Algebra System (CAS) software. CAS are computer software packages which have incorporated all the rules and methods of ``pen and paper" mathematics into a computer program. Hence, all the pages and pages of complex and routine algebra that students are required to perform can now be done on a computer by merely entering the relevant commands into the CAS software. With CAS becoming more accessible, some educators have questioned the need to teach (old fashioned) ``pen and paper" analytical mathematics. Why learn to memorize all these mathematical ``recipes" when all one needs to do is to type the equation into a computer and instantaneously obtain the solution? This is an important question to raise but the value of analytical mathematics must not be dismissed. Analytical mathematics nurtures critical and logical thinking skills in engineering students. These are very important skills for engineers to possess. But using CAS in the teaching of mathematics adds a new dimension to the subject. The added value of incorporating computer graphics and animations available in CAS to help students understand intricate maths ideas is unquestionable. The main benefit of CAS is not to replace the didactic lecturer. In fact, most educators use CAS to enhance their lectures. CAS are typically used in a ``mathematical laboratory" environment where students are asked to work in teams to solve short assignments designed to give students a deep understanding of complex but important ideas in mathematics. It has been found by many engineering educators that performing exercises in laboratory sessions provide students with a healthy grasp of complicated mathematics concepts that are of paramount importance to engineering students.

In light of the discussion above, this paper will address the following three issues in the teaching of mathematics modern engineering undergraduate degree programs.

- How much mathematics and the sequence in which mathematics should be taught in our undergraduate engineering programs.
- Explore how *computational* mathematics can/should be incorporated in the engineering math curriculum. *Computational* mathematics is important to engineers as it provides the required knowledge to use commercial engineering software intelligently.
- The use of Computer Algebra Software (CAS) package in the teaching of engineering mathematics.

Structure of the Mathematics Education in Engineering Curriculum

Mathematics has traditionally been taught as a separate subject in engineering courses around the world. This is because mathematics is common across all engineering disciplines and it was thought that teaching and learning would be more efficient if students from all engineering disciplines were to be grouped together to learn mathematics. At first glance, this might seem like a logical approach but it raises a few issues. First and foremost, by isolating mathematics from engineering subjects, it is difficult for students to see the mathematics ``in context". As engineering students, they would like to see how mathematics is used to solve practical engineering problems. However, if maths is taught as a separate subject, then the starting point of reference is usually the mathematical equation ..... not the underlying engineering problem. Thus, many students fail to see the relevance of learning mathematics. Another issue that needs to be considered is the mathematics requirement of different engineering disciplines. Different engineering disciplines require knowledge of different maths topics and the level of knowledge in each math topic is also different for each engineering discipline. Thus, if all engineering students were required to undertake common maths subjects, then it is very likely that they will be learning mathematics that they will not need in their entire engineering career. Another disadvantage of teaching mathematics subjects to all engineering students is that the class sizes are usually very big. This, together with mathematics usually being taught in a didactic manner, (with a lecturer teaching a few hundred students in a big theatre) is an undesirable situation as it is not conducive for creating a good learning environment.

In summary, the reasoning behind isolating the teaching of mathematics from engineering science might seem logical from an administrative perspective, but lacks pedagogical benefits as far as students are concerned. Using this argument, it would seem logical to simply (naively) incorporate the teaching of mathematics into the teaching of engineering science subjects of the different disciplines of engineering. However, this would not be an optimal solution for the following reasons.

- Many maths topics are needed across many engineering disciplines. Thus, if different lecturers teach the same maths topic in different engineering subjects, it could create a situation where there is repetitive teaching in many engineering subjects. This would simply be a waste of time and resources.
- There is a real danger for students to think that a particular maths topic, say the solution to ``ordinary differential equations", is only relevant to that particular engineering subject that it is taught. For example, it is difficult for Mechanical engineering students to realise that the solution to ``ordinary differential equations" is also important for Electrical Engineers if ``ordinary differential equations" is taught in a Mechanical engineering subject.

Despite these concerns there have been efforts to increase students engagement with mathematics by incorporating it into engineering science subjects. Subjects using this strategy have been developed and offered by the Center for Innovation in Engineering Education (CIEE) at Princeton (Brown (2005)). The goal of the integrated (engineering and maths) approach can be summarise in the following statement from Prof. Jennifer Rexford, a computer science professor at Princeton

``The goal of the integrated course is to give BSE freshmen early exposure to engineering and to learn how math and physics are associated with it. Traditionally, these subjects are taken separately and most students do not have much exposure to the discipline of engineering until their sophomore year."

Thus, the idea of incorporating mathematics into the teaching of engineering subjects is relatively new but it has strong support by educators in elite engineering schools. However, the program initiated in CIEE is still relatively new and there are still questions if this integrated (engineering and maths) approach will meet the desired outcome of motivating students to gain a deeper understanding of mathematics.

Another point for consideration is how much mathematics is actually needed by each of the engineering disciplines. Currently at the University of Melbourne, all engineering students (of all engineering disciplines undertake the same maths subjects in 1st and 2nd year. As a result, engineering students might be forced to learn maths topics that they might not actually ever need in their chosen engineering discipline. For example, there is a maths topic called ``partial differential equations" that is critical for Chemical and Mechanical engineers but Software engineers need not know this topic at all .... possibly in their entire career. There are many engineering courses at elite institutions that do not require students for all engineering disciplines to undertake exactly the same maths program. For example, the contrasting nature of the maths program for Civil and Electrical engineering students at Stanford University. The Electrical engineering students do about twice as many maths subjects as the Civil engineering students. Princeton also seem to design their maths program to be tailored to a particular engineering discipline.

Teaching of Computational Mathematics

The mathematics that is normally taught in engineering courses in the past can be broadly categorized as *analytical* mathematics where maths problems are solved ``by hand''. Besides teaching engineering students how to solve math problems, one of its other main strength is that it helps develop generic skills in logical and critical thinking. The main drawback with *analytical* techniques is that these methods can only be used to solve math problems that are relatively ``simple'', problems that have been significantly simplified from ``real world'' applications. Thus, the solutions obtained using *analytical* techniques might not be useful for the practicing engineer. There is now an emerging branch of mathematics called *computational/numerical* mathematics, which is the theory that gives rise to computer algorithms that can be used to solve very complex math problems. The problems that can be tackled using *computational* mathematics are typically closer to ``real world'' problems. Computer algorithms generated using *computational* mathematics are implemented in many commercial software that are used by many practicing engineers in the workforce today. Thus, it is critical for engineers to know and understand the *computational* algorithms that are used in the software that they interact with on a daily basis.

Many academic institutions are starting to introduce *computational* mathematics into their undergraduate engineering degree program. Because *computational* mathematics is quite different to *analytical* mathematics, questions have been raised on how *computational* mathematics can be incorporated into the engineering course structure. Should *computational* mathematics be taught as a separate maths subject? Or should it be incorporated into the teaching of an existing *analytical* math subject? In the University of Western Australia, *computational* mathematics is taught as a separate maths subject called ``Modelling and Computer Analysis for Engineers'' (CITS2140). This subject concentrates solely on developing the computational algorithm required to solve mathematical equations and does not teach techniques in *analytical* mathematics. Teaching *computational* mathematics as a separate subject also occurs at the Mechanical engineering program at the University of Melbourne in a subject entitled ``Systems Modelling'' (436-204). A slightly different approach has been taken at Stanford University. In some of the Mechanical and Chemical engineering programs, students are recommended to take subjects called ``Vector Calculus for Engineers'' (CME100), ``Ordinary Differential Equations for Engineers'' (CME102) and ``Linear Algebra and Partial Differential Equations for Engineers'' (CME104). These subjects teach students how to solve mathematical equations using **both** *analytical* and *computational* techniques. This approach introduces the use of computers in the teaching of mathematics and makes the maths more graphical and fun for more students. As most engineering students are visual learners (i.e. they acquire a better understanding of a particular topic if it is visually communicated to them (Felder and Silverman, 1988)), combining the two forms of mathematics seem to make a lot of sense.

The algorithms generated from *computational* mathematics will only be useful if they are implemented into a computer program. 20 years ago, scientists and students used to write computer programs mainly in Fortran. More recently, the C and C++ languages are becoming more popular. By far, the most popular programming language for *computational* mathematics at the moment is **MATLAB®** (http://www.mathworks.com). Its ease of use and simple tools for debugging has made **MATLAB®** a popular programming language for scientists, academics and students.

Computer Algebra Software

Computer Algebra Software (CAS) are computer software packages that can perform *analytical* maths calculations. All the complicated symbolic manipulation that students learn in high school can now be automated with CAS. Some of the more popular CAS packages are **Maple** (http://www.maplesoft.com), **Mathematica** (http://www.wolfram.com/), **Mathcad** (http://www.mathcad.com/) and **Derive** (http://www.chartwellyorke.com/derive.html). One other feature of CAS is its ability to provide advanced graphical illustration of complicated mathematical solution. This makes it enormously easier for engineering educators to perform exercises and apply mathematical subject matter to engineering problems. Garcia *et al.* (2005) has found that these entities are especially useful and appropriate for teaching and learning purposes. A study conducted by Comacho and Depool (2002) found that the appropriate use of CAS helps motivate student to learn and understand mathematics. The role of engineering educators is to encourage students to use these tools to understand the full extent of the ability of mathematical models to solve ``real-life" engineering problems.

The most extensive documented use of CAS is the Connected Curriculum Project (CCP) (see http://www.math.duke.edu/education/ccp/). This project has developed a collection of interactive CAS software modules which can be used in mathematical laboratory sessions. These sessions are two hour long that encourages student to work in small groups in order to solve mathematical problems and acquire a deep understanding of complicated mathematical concepts. Garcia *et al.* (2005) has found that using CAS helps bring about positive changes in which mathematics is typically taught. Without the use of CAS, mathematics classes are taught in a didactic manner, where the professor/tutor tends to be the sole centre of attention. When CAS is incorporated into the mathematics curriculum, there is an observable increase in student participation, autonomous activity and interaction among students, thereby making the process of acquiring and constructing mathematical knowledge more student-centred. The role of the lecturer/tutor is to act like a facilitator who encourages collaborative and discovery learning. Due to the interactive nature of these CAS tools, students are able to attain a higher level of abstraction in mathematical problem-solving (Garcia *et al.* (2005))

In order for the laboratory classes to be conducted efficiently, lecturers/tutors must be trained to facilitate student interaction. Winter *et. al.* (2001) addresses potential problems of student/instructor in mathematics laboratory classes and suggest ways to overcome obstacles in this process. In the CCP program, lecturer/tutors are instructed to facilitate the sequence of themes that is used in CCP modules. These themes are experiment, analyze, justify, and generalize. The process encourages students to discover mathematics through exploration, write mathematics to explain their observations and make predictions with mathematical models. The computer laboratory provides a discovery-based learning environment that mimics the experiences many students have in their science laboratories, providing a view of mathematics as a science Fitchett (2002). In a study conducted by Hannah (2001), it was found that one of the major advantages of the CCP approach is that it lets students explore patterns and symmetry in mathematical expressions easily. The graphical power of the CAS also enables students to quickly and graphically view solutions and illustrate difficult and abstract mathematical concepts easily and effectively.

Even though it would seem like CAS has been quite successfully been implemented in many mathematical curriculum at University and high school levels, there were initially many obstacles in trying motivate math educators to incorporate CAS in their teaching. Austria was one of the first countries to recognise the potential of incorporating CAS and they purchased a nation wide license for the CAS package **Derive** in 1990. But rate of adoption by educators was poor due to insufficient training and teachers not experience in using CAS in a classroom. In addition, it also requires student to be efficient in using CAS. Learning a CAS package such as **Maple** could be difficult for some students and they can get frustrated in having always to remember to type in the complicated computer syntax before the CAS can give you the solution. There is also the issue of not having enough computer resources available to run the CAS (Schneider (2000)). However, since then, teachers have had much more training in technology and computers themselves have gotten a lot faster. Thus, since 2000, the rate of adoption of CAS into mathematical curriculum has increased dramatically in many universities.

Because of the recent interest in CAS, there have been many studies on how to best use CAS in the teaching of mathematics. Dana-Picard (2004) investigated the role of CAS in the teaching of engineering mathematics. In that article, the author advocated the use of ``low-level" CAS commands in the teaching if engineering mathematics. These ``low-level" commands allow students to concentrate on the underlying mathematical ideas, without having to worry about making mistakes in complex algebraic manipulations (Rielly (2004)). ``High-level" software commands can be used to obtain quick answer. But these commands treats the underlying mathematical ``engine” as a black box. The students do not know what is going on ``behind the scenes" after they press the button on the computer. Lopez Molina (2005) used CAS tools and suggested how they can be used to give students a better understanding of the behaviour of solutions of ``partial differential equations". An investigation carried out in Norway (Hornaes (2000)) showed that students believed CAS is more appropriately used in the teaching of engineering science subjects, rather than in purely mathematics subject. This finding thus supports the view that CAS is most appropriately used in subjects that incorporates the teaching of mathematics inside an engineering science subject.

Concluding comments

This paper illustrates the many issues that must be considered when designing a good mathematics curriculum in an undergraduate engineering program. A review of the maths curriculum in many elite engineering schools world wide has shown that different universities have different methods of incorporating maths into their engineering program. Some universities have different maths curriculum for different engineering disciplines but other universities forces students of all engineering disciplines to undertake the same maths program. Naturally, different engineering disciplines have different levels of mathematical requirements. So the question that needs to be asked is:

Do engineers need to study maths that they do not need in their career?

The naive answer would be “no”. But this is a very complex issue as it is undeniable that the development of the logical and critical thinking process through performing maths calculations (whether related to their chosen engineering discipline or not) is priceless in the training of any engineer.

In general, mathematics can be divided into two different branches, *analytical* and *computational* mathematics. Previously, the teaching of mathematics at universities has mainly concentrated on *analytical* mathematics. However, with the advancement of modern day technology, there is no denying that computational mathematics has become more and more important in the lives of practicing engineers. Thus, there is now an urgency for engineers to know more about *computational* mathematics. The question that must be asked is, how much more?

With the advancement of computing technology, some engineering educators have effectively used Computer Algebra Software (CAS) packages in their teaching of analytical mathematics. CAS is typically used in a laboratory environment where has been shown to increase students engagement with mathematics. Using CAS in a laboratory environment also encourages collaborative and discovery based learning where students can gain a deeper understanding of mathematics.

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