Puzzle-Based Learning: The first experiences

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***Abstract**: While students of science and technology are trained to recognise familiar problems with known solutions, they may not be sufficiently prepared to address novel real-world problems. As a step in this direction, we have created and experimented with a new course that is aimed at getting students to* think *about how to frame and solve* unstructured *problems. The pedagogical goal is increase students' mathematical awareness and general problem solving skills by employing* puzzles*, which are educational, engaging, and thought provoking. Over the past year we have run a new course on Puzzle-Based Learning in Australia, United States, and Qatar, with classes from 15 to 380 students. The course is offered in two versions: a full-semester course, and a unit within a general course, such as* Introduction to Engineering*. In this paper we share our experiences in teaching both courses and discuss our pedagogical objectives, teaching approaches, and coursework. Preliminary observations indicate that the puzzle-based learning approach is assisting students by providing a framework to explore critical thinking, as well as being fun and interesting.*

Introduction

Students often have difficulties in independent thinking or problem-solving skills regardless of the nature of a problem. At the same time, educators are interested in teaching "thinking skills" rather than "teaching information and content." The latter approach has dominated in the past. As Fisher (2001) wrote: "*… though many teachers would claim to teach their students 'how to think', most would say that they do this indirectly or implicitly in the course of teaching the content which belongs to their special subject. Increasingly, educators have come to doubt the effectiveness of teaching 'thinking skills' in this way, because most students simply do not pick up the thinking skills in question*."

Further, many analysts lament the decreasing mathematical skills of students. A recent *Mathematics Working Party Final Report*, issued by the University of Adelaide (June, 2008) includes statements such as "*There is an urgent need to raise the profile and importance of mathematics among young people…"* and *"The declining participation in mathematics and related subjects is not limited to Australia…".*

The University of Adelaide and Carnegie Mellon University have introduced a new course titled Puzzle-Based Learning to address all of the issues raised here.

The puzzle-based learning approach

The puzzle-based learning approach aims at encouraging students to *think* about how to frame and solve *unstructured* problems - those that are not encountered at the end of some textbook chapter. Our goal is to motivate students, and also to increase their mathematical awareness and problem solving skills by discussing a variety of *puzzles*. The course is based on the best traditions introduced by Gyorgy Polya (1945) and Martin Gardner (1961) over the last 60 years.

In this course we concentrate on *educational puzzles* that support problem-solving skills and creative thinking. These educational puzzles satisfy most of the following criteria:

1. *Independence*: The puzzles are not specifically tied to a single problem-solving domain.
2. *Generality*: Educational puzzles should explain some universal mathematical problem-solving principles. This is of key importance. Most people agree that problem solving, like any other skill, can only be learned by deliberate practice, i.e., by solving problems. However, this activity must be supported by strategies provided by an instructor. These general strategies would allow for solving new yet unknown problems in the future.
3. *Simplicity*: Educational puzzles should be easy to state and easy to remember. This is also very important, as easy-to-remember puzzles increase the chance that the solution method, including the universal mathematical problem-solving principles, is also remembered.
4. *Eureka factor*: Educational puzzles should initially frustrate the problem-solver, but with the promise of resolution. A puzzle should be interesting because its result is counter-intuitive: problem-solvers often use intuition to start their quest for the solution and this approach can lead them astray. Eventually a *Eureka!* moment is reached (Martin Gardner's *Aha!*), when the correct path to solving the puzzle is recognized. The *Eureka* moment is accompanied by a sense of relief, the frustration that was felt during the process dissipates and the problem-solver may feel a sense of reward at their cleverness for solving the puzzle. The Eureka factor also implies that educational puzzles should have elementary solutions that are not obvious.
5. *Entertainment factor*: Educational puzzles should be entertaining and engaging. Entertainment is often a side-effect of simplicity, frustration, the Eureka factor, and an interesting setting.

Educational puzzles can play a major role in attracting students to computer science and engineering programs and can be used in talks to high school students and during open-day events. Puzzles can also be a factor in retaining and motivating students. Further, there is a strong connection between the ability to solve puzzles and the ability to solve industry/business problems (Poundstone, 2000).

The puzzle-based learning courses: types, structure, and content

There are a few different versions of the puzzle-based learning course being taught currently. The course can be offered as a full-semester (three units) elective course or freshman seminar (typically 3 contact hours per week), one unit freshman seminar or one unit core module as part of other courses.

One of the important points about puzzle-based learning courses is that the course is not about presenting and discussing a variety of puzzles but rather about presenting, discussing, and understanding problem-solving principles and some mathematical principles in the context of puzzles that serve as entertaining illustration of the presented concepts. Also, the process of understanding problem-solving principles leads students through a variety of topics, exposing them to many important concepts at early stages of their college education.

Despite a variety of possible offerings of puzzle-based, the structure of the course is very much the same. The topics listed below correspond to weeks of a 12-week:

1. **Introduction:** What it is all about?
2. **The problem:** What are you after?
3. **Intuition:** How good is it?
4. **Modeling:** Let's think about the problem
5. **Some mathematical principles:** Do you see it?
6. **Constraints:** How old are my children?
7. **Optimization:** What is the best arrangement?
8. **Probability:** Coins, dice, boxes, and bears
9. **Statistically speaking:** What does it mean?
10. **Let's simulate:** Can we generate the answer?
11. **Pattern Recognition:** What is next?
12. **Strategy:** Shall we play?

For more information on the nature of puzzles and the approaches used in puzzle-based learning, readers are directed to the website associated with the text, www.PuzzleBasedLearning.edu.au. We now discuss two of the implementations of this course, at the primary development site, The University of Adelaide and at Carnegie-Mellon University, Pittsburgh.

A Sample Puzzle

One of our favourite puzzles to encourage students to think is the classic Monty Hall problem. In this simple problem there are three doors, one of which hides a prize and two others conceal nothing. The host asks you to choose a door. After you have done so and indicated your choice, the host opens one of the other doors that you did not pick and which does not contain the prize (the host knows the placement of the prize, of course). Then the host offers you the chance to change your original door pick to the remaining unopened door (i.e. the door not picked initially by you nor opened by the host). Should you do so and change? Many students will see no advantage in changing doors, while some will argue that they now have a 50% chance of being right. The correct answer, that you should always change doors as it doubles your chances of winning the prize, is introduced to the student through discussion, demonstration and active participation. It also clearly illustrates the value of thinking about your assumptions, being cautious about your intuitions, the utility of qualitative reasoning and quantitative reasoning, and reasoning with a problem variation.

First experiences: The University of Adelaide

The initial implementation of puzzle-based learning was a 1-unit course set as a component of a 3-unit first-year course, *Introduction to Electronic Engineering*. From 2009, this 1-unit offering was placed into introductory courses across the engineering programs offered within the University. A 3-unit first-year course for students planning to major in computer science was launched simultaneously in 2009, and made available to all non-engineering students in the University. We refer to the 1-unit offering as PBL-E (PBL for Engineers) and the 3-unit offering as PBL Main. The courses cover the same material, at different levels of depth and a student will take either the 3-unit offering as a separate course or the 1-unit offering as part of another introductory engineering course. The intake for the two courses is quite different, as PBL-E students have a higher Tertiary Entrance Rank on average and also have two courses of mathematics from secondary school. The students in PBL Main may have a single course of mathematics, if enrolled in the Bachelor of Computer Science, or may have no mathematics beyond the middle of Secondary School, if from other programs.

In 2008, 325 students undertook the first offering of 1-unit PBL, with a weekly 1-hour lecture, supported with on-line forums and assessed through weekly assignments and a final examination of 1-hour duration. In 2009, 370 students were undertaking the 1-unit PBL course, with 102 students undertaking the 3-unit PBL course. The PBL-E course remained essentially the same in structure but the 3-unit course added an additional lecture per week and weekly tutorials. This allowed the development of the material in further depth. The majority of PBL Main homework was composed of two questions to be completed in the week, rather than the single question of PBL-E.

Lectures in PBL follow a set pattern. The first lecture of the week presents the solution to the previous homework, identifies the key points for this week's lectures and then builds on the topic area. The lecture concludes with the next assignment. Lectures are recorded and the lecture slides, recordings and all assignment work are available on the course electronic forum. The electronic forums also provide message boards for student interaction. PBL Main has a second lecture that develops the themes of the week's topic. Lecture materials are developed in parallel, with the single PBL-E lecture derived from a revision and abridgement of the two PBL Main lectures for that topic to maintain currency between the two courses. Each topic is illustrated by a variety of puzzles presented in an interactive manner. The course introduces a few simple problem-solving rules that we refer to in every class. Every week students are presented with homework assignments: one or more puzzles on the topic covered in the class. The following week, at the beginning of a class, the solutions are presented and discussed. Homework contributes 30% towards the final grade, and the final exam contributes the remaining 70% of the final grade. Students have access to all lectures slides, audio lecture recordings, and additional material, including course software.

Tutorials are offered for PBL Main and allow students to take part in collaborative problem solving exercises, with a tutor to provide assistance and guidance. Tutorial groups are up to 25 students, with sub-group formation of 5-8 students for problem solving. During these sessions, we introduce fundamental mathematical concepts that are useful in the later course, including counting and the bases of probability, including factorials, combinations and permutations. This addresses the difference in mathematical preparation that was identified in the intake. While a good grasp of mathematics can be useful for PBL, it is not essential. Problem specification has been of key concern to us, as the larger classes contain a percentage of students who are accustomed to a completely specified problem and are uncomfortable when confronted with problems that are not complete specified or, in the student's opinion, are not sufficiently and exactly specified. While some students regard this as a challenge, and an intellectual freedom, other students find this to be a stumbling block.

Assessment of the course has proven to be one of the largest implementation issues. Students are interested in the material but their interest can easily be capped when they feel constrained by the assessment mechanisms, or feel that they haven't received sufficient, personalised, feedback. Early assessment for PBL-E revolved around a mark for each assignment out of 5, followed by feedback to the group demonstrating the marking scheme and the solution. The solution was also presented at the start of the lecture that corresponded to the hand-in time, to allow students to immediately gain feedback on the quality of their solution. PBL-E's student numbers posed a significant resource issue, as it takes approximately 2-3 minutes to mark each assignment, without detailed feedback. Thus, marking load starts at approximately 8 hours for each assignment and a team of markers have had to be employed and trained for consistency of response. PBL Main has a much smaller enrolment but employs detailed, personalised feedback, which also takes approximately 8 hours for a week's assignments. The requirement for a consistent and reproducible marking scheme that can be assigned to multiple markers constrains the range of problems that can be offered, as problems with too many possible solutions become effectively impossible to mark across 370 students. In response to this, we have considered many alternatives and are currently developing problems that may have multiple possible solutions, all of which may appear to be correct when, in fact, only one is. Again, this is an issue of problem and solution specification. Controlled use of multiple-choice questions, with between 8 and 10 options, allows markers to quickly identify the flaw in reasoning and mark the student. We are also investigating the possibility of reducing the dependency on mark-based assessment.

Early student response shows that students enjoy the course material and that it does develop their thinking skills, but a number of students, especially in PBL-E, encounter issues with the assessment model and perceived lack of feedback. For some students, their perception of the assessment mechanism can develop a very negative and unproductive approach to the course. We are actively seeking to address this, by allocating more resources to marking and feedback and through the use of automated marking mechanisms that allow more rapid response. Future implementations of PBL-E may include tutorials, or alternative assessment mechanisms.

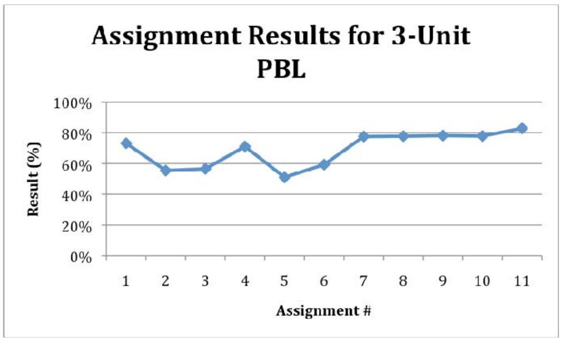

Figure 1a: Student results for assignments over the Semester

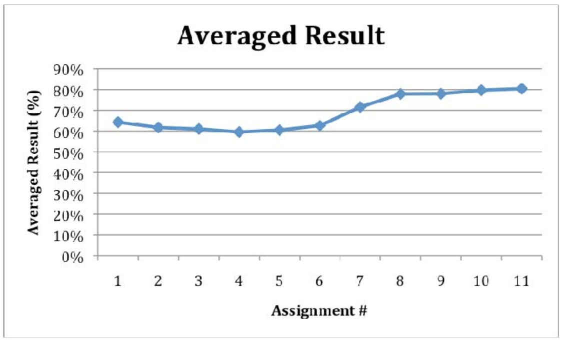

Figure 1b: Averaged student results for assignments over the Semester.

The first implementation of PBL as a 3-unit course has shown a consistent development of student puzzle-solving skills, culminating in excellent examination performance that outstripped our initial expectations. Figure 1 shows the overall improvement of the students during the semester, as we provided personalised feedback as well as overall assignment solutions. Assignment 1 had a simplified and generous marking scheme, and assignment 4 contained questions designed to encourage student participation and was less rigorous than assignments 2, 3 and 5. Figure 1a clearly shows the improvement over time for the cohort and the same number of students was submitting solutions at assignment 11 as were submitting at assignment 1. There was a small drop in submissions for assignment 12, as the problem was placed at the end of the semester and was more challenging than previous problems. Figure 1b shows the general trend in improvement, where the value shown at a given assignment is the average of the assignment result and those on either side.

The exam performance for this course was also excellent, with six previously unseen puzzles presented to the students. The overall failure rate for the course was 19%, with a failure rate in the examination itself of less than 10%. Actively participating students passed and passed well. The assignments results for the 1-unit Engineering Course did not show the same steady improvement, although the moving average more clearly shows the improvement over time. Again, the student performance begins to stabilise after the midpoint of the course.

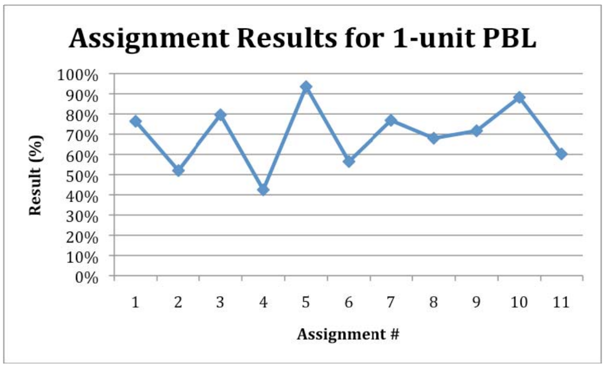

Figure 2a: Assignment Results for the 1st Semester 1-unit PBL course.

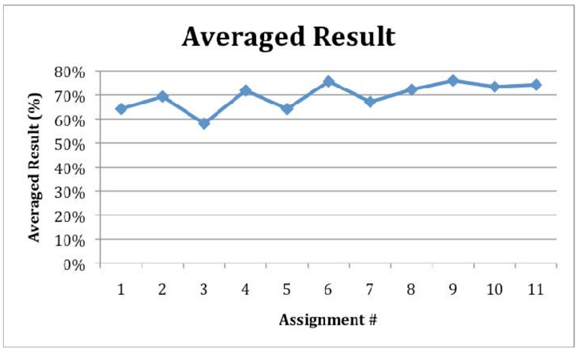

Figure 2b: Averaged assignment Results for the 1st Semester 1-unit PBL course.

Once individualised feedback was introduced in later assignments, the overall performance improved. However, the final assignment was both challenging and relatively poorly done. The number of submissions stayed consistent throughout the course, with a slight decrease for assignment 11 due to end of semester and, potentially, the level of challenge. This 1-unit course was part of larger courses in introductory engineering and, because of this, failure rates in the final course do not correspond to failure rates in the PBL component. Within the PBL component, the overall failure rate was 28% for the course, with an examination failure rate of 27%.

We believe that the two lectures a week in the 3-unit course, combined with weekly tutorials, have greatly assisted students in developing their puzzle-solving skills and are working to incorporate the lessons learned from this semester into the next teaching semester for these courses.

First experiences: Carnegie Mellon University

At Carnegie Mellon University, Pittsburgh, puzzle-based learning was offered as a 9 unit (3 credit) Freshman Seminar in spring 2009. Given the seminar nature of the course, enrolment was capped at 15 but it was encouraging to see that the wait list was longer than the class enrolment! The class had an inter-disciplinary mix of students majoring in Information Systems, Computer Science, Psychology, Statistics, Cognitive Science, Economics, and Physics. The class met twice a week for 80 minutes. In addition to what is described above in the Adelaide experience, given the smaller size of the class we were able to experiment with several alternative themes. For example, after the introductory classes, each session started with a puzzle-of-the-day. One student would present a puzzle of their choice. The class as a whole would try to solve the puzzle with hints and guidance provided by the puzzle poser. Student chosen puzzles ranged across the gamut of logic puzzles to diagrammatic reasoning to physical puzzles (tangrams). Students had to submit a one-page write-up of their puzzle, solution, and more importantly, their reflection on the puzzle: what did they find interesting in the puzzle, variations, how does the solution tie into the general class discussions etc. In addition to the puzzle-of-the-day, we also conducted a *puzzlethon* where students again presented a puzzle of their choice. But this time the class voted on the best puzzle (a combination of presentation plus the nature of the puzzle) and prizes were distributed.

During our discussion of scientific induction and mathematical induction, given the smaller size of the class, we played Robert Abbott's inductive game of Eleusis (Gardner, 1997) that models the process of scientific method. To introduce students to some of the thoughts of leaders in the arena of problem solving we watched a few videos. These included Polya's "Let us teach guessing" wherein Polya beautifully illustrates several problem solving heuristics (that are embraced by puzzle-based learning) in the process of deriving a solution to the 5-plane problem; an interview with Herb Simon on being a researcher with advice to undergraduates; Richard Feynman on problem solving and induction; and Will Shortz's documentary "Wordplay" on crossword puzzles, their creators, and solvers. We also visited the Pittsburgh Super Computer Center open house to get a glimpse of problem solving in the real world. To emphasize the link between the thought processes involved in solving puzzles and addressing open real-world problems we examined a few case studies including the recently cracked Netflix Prize (www.netflixprize.com) and the classic work of Mosteller (1984) in resolving the authorship of some of the disputed Federalist papers.

Student evaluation was done with components for class participation, puzzle presentations, homework assignments, three exams and a few in-class quizzes. It has been gratifying to see that the response to the class has been favorable with some students commenting it was the best class they had that semester. A similar version of this course was offered at Carnegie Mellon University in Doha, Qatar in summer 2009. The freshman seminar is also slated to be offered in Pittsburgh in spring 2010.

Lessons Learned & Future Plans

Puzzle-based learning is an experiment in progress. The goal is to foster general domain independent reasoning and critical thinking skills that can lay a foundation for problem-solving in future course work. As fun as puzzles inherently are, they are just a means to this pedagogical end. Our preliminary experience in different contexts has been encouraging and well received as we continue to explore this approach. As instructors for this new course it has been a learning experience for us also. Based on our preliminary experience some of the pedagogical lessons for us have been:

- The value of separating problems for in-class discussions, homework, and exams.
- Showing students alternative modes of reasoning for the same puzzle.
- Collecting puzzles to illustrate various problem solving strategies. For example: understanding the problem (the backpackers puzzles); iterative solution (5 pirates puzzle); ambiguous problem specification (determining number of 0s in 100!); collaborative problem solving (broken squares); analogical / case-based reasoning (inter-twined ropes)
- The development of a hierarchy of problem solving: puzzle-based/problem-based/project-based. We would like to take the ideas from this foundational course and more explicitly show how these puzzle solving strategies can map on onto real life problems.
- Computational thinking (Wing 2006) is an evolving model helping to frame what computer science is about. CT and PBL share many of the same pedagogical goals, which we plan to explore further.

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