

# Mathematical integration throughout the BE: lecturer expectations versus student knowledge

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### BACKGROUND

Students who perform well in high school mathematics often struggle at university with mathematical integration. Failure rates of 30-35% are common across first-year mathematics courses at The University of Queensland, with integration a threshold concept most students do not understand well even at the end of first-year. Do students who perform well at school somehow lose their integration knowledge between the end of high school and the start of university some four months later? Or did they actually not grasp integration at school yet somehow manage to achieve good grades?

In order to assess the knowledge of incoming students and thus provide a benchmark against which their performance at university could be assessed, Kavanagh et al (2009) analysed the results of a competency test delivered prior to Semester 1. Whilst there were many positive outcomes of the competency test, which is now implemented every year by a number of national institutions, it highlighted mathematical integration as a difficult concept that many students struggled with. This difficulty was anecdotally reported to occur across the four-year degree program.

#### PURPOSE

This research identifies the types of integration that are used throughout the engineering degree program, and academics' perceptions and expectations of students' integration skills. It forms the prelude to a teaching and learning initiative to address any deficit in students' ability to understand and apply mathematical integration to a variety of problems.

#### **DESIGN/METHOD**

Data on integration techniques used in courses across the four-year engineering degree program were collected via a survey. The same survey was used to obtain lecturers' perceptions of the level of students' integration skills, and whether there would be value in an online competency test for each integration technique used prior to their course(s). The second phase of the research (not reported here) will support students with appropriately timed tests and revision exercises.

#### RESULTS

Integration skills were found to be required across all years and disciplines, with a recurring theme of one-dimensional concepts. Lecturers at *all* year levels and across *all* disciplines thought that students' integration skills were either poor or, at best, satisfactory. Actual knowledge, assessed by the pre-semester competency test and end of semester examinations, supported this. Lecturers of higher level courses were more likely to rate skills as 'poor'. Interestingly, of those who said 'satisfactory', the majority taught courses where not much integration was used.

#### CONCLUSIONS

Mathematical integration is a fundamental skill that all engineering students are required to use throughout their undergraduate and professional lives. Therefore it is important that any deficit in this skill be addressed. The next phase of this research will build on the work of Kavanagh et al. (2009), and will see integration competency tests administered before key courses in order to allow lecturers to assess the incoming cohorts' knowledge level, and also to allow students to revise knowledge before the course begins.

#### **KEYWORDS**

Mathematical integration, skill mapping, knowledge revision, diversity

# Introduction

There are substantial and ongoing concerns in the Australian and international tertiary education sectors about students' transition from secondary to tertiary mathematics. Declining enrolments in university mathematics and increasing failure rates in first-year are often attributed to falling participation in advanced mathematics in secondary school, and less stringent university entry requirements have adversely affected students' mathematical preparedness for university study, in particular engineering.

This appears to be the case at The University of Queensland (UQ) where failure rates of 30 - 35% are common across first-year mathematics courses. Mathematics lecturers note that the concept of mathematical integration, supposedly learnt at high school, is problematic knowledge for many of these students. Lecturers into engineering science courses, such as basic engineering mechanics, as a matter of course provide revision resources for integration at the beginning of semester, indicating to students that this is required and assumed knowledge.

The paper begins with an overview of what is known about teaching mathematical integration in secondary and tertiary institutions and then outlines the integration topics taught in Queensland high schools and at UQ. It finishes with an exploration of the use of specific concepts across the engineering degree program as well as lecturers' perceptions of students' abilities.

# Research on calculus teaching and learning in secondary school

In Queensland calculus forms a considerable part of the senior intermediate mathematics syllabus with approximately 100 hours out of 220 over two years devoted to this topic (integration by itself is approximately 35 hours). If pre-calculus topics such as functions are included then it is even more. However, diagnostic testing and first-year examination results reveal that many university students remember, or understand, little of their Grade 11 and 12 mathematics in comparison to topics they studied in primary and early secondary school (Jennings, 2008, 2009, 2011; Kavanagh et al., 2009).

Numerous researchers (Orton,1983a, b; Schoenfeld, 1985; Hiebert and Lefevre, 1986) found that students often do mathematical questions with little or no understanding of the way they are actually doing them. Oaks (1987/1988, 1990) went further, claiming that some students do not even realise that there are concepts underlying the methods they use. Such students believe that there is no meaning in mathematics; rather mathematics is all about performing pointless operations on meaningless symbols, and that rote memorisation, by both student and teacher, is how people learn mathematics (Oaks, 1990). Berry and Nyman (2003) found that university students who had studied calculus at school understood derivatives and integration only in terms of their algorithmic rules and that they had not developed "an appreciation of the theoretical concepts or an intuitive 'feel' for the idea" (p. 481). They asserted that students' concept construction often occurs by accident which leads to intuitive or naïve structures that can be very resistant to change.

Teachers can and do affect the way a student learns. Tall (1991) asserts that many teachers and mathematicians make the same error when they teach calculus for the first time: they try to simplify a complex mathematical topic by breaking it up in smaller parts that can be ordered in a sequence that is logical from a mathematical point of view. From their viewpoint these smaller parts may be seen as part of a whole; however, the student may see the pieces as they are presented, in isolation, 'like separate pieces of a jigsaw puzzle for which no total picture is available.' Tall (1991, p. 17).

It is also possible that assessment in secondary schools, often an externally set examination which focusses more on procedures than concepts (Berry and Nyman, 2003), does not increase a student's grasp of mathematical integration. Typical calculus assignment and examination questions are 'solve, sketch, find, graph, evaluate, determine, differentiate,

integrate' (Ferrini-Mundy and Guenther-Graham, 1991). A detailed investigation of exams from UQ's advanced mathematics bridging course and first-year calculus and linear algebra course over the last ten years supports this observation. Whilst sketching and graphing questions can elicit conceptual understanding, the other activities focus on procedures.

As university lecturers we have little opportunity to change these practices at the school level (we can at university) but it helps to know why our students have a poor understanding of mathematical integration. Another factor contributing to the supposed poor understanding is time. Thirty-five hours of integration at secondary school may seem significant (in fact, it is nearly the same length as a standard university course); however, it is spread over one and a half years and not taught continuously. Combine that with a long break between the end of secondary school and the beginning of university, and perhaps the expectations of university lecturers are too high.

# Research on calculus teaching and learning in university

There have been many studies into the most effective way to teach calculus to university students. The outcomes of these studies showed that:

- using technology to help visualise calculus, although harder for students than traditional methods, was more effective (Habre and Abboud, 2006);
- teaching calculus through applications (e.g., heat removal by fins made from different metals) allowed students to gain a better understanding of the core calculus concepts (Young et al., 2011);
- overall secondary school academic achievement, secondary school mathematics achievement, and current university academic performance were statistically significant in affecting a students' performance in university calculus (Buchalter and Stephens, 1989);
- a poor conceptual understanding of calculus can be correlated with a lack of critical thinking and problem solving skills as well as a superficial depth of knowledge (Engelbrecht et al, 2007); and
- first-year test workshops and support sessions can reduce failure rates in second-year mathematics courses (Cuthbert and Macgillivray, 2003).

While there have been many studies into high school and university students' calculus understanding, it appears no longitudinal study from high school to university has been undertaken. One of the authors is currently investigating how students' knowledge and understanding of calculus develops over a three-year period, from Year 11 through to first-year university in order to provide an insight into students' abilities in calculus.

### What should students be able to demonstrate after one year of uni calculus?

A survey of engineering academics across all disciplines at UQ conducted in 2010 prior to a major review of first-year engineering showed that all the currently taught mathematics topics were of value and required for second-year engineering. However, a survey of 24 'calculus experts' from America found that there was only one calculus topic, derivatives, that everyone agreed students should be able to do upon completion of first-year calculus (Sofronas et al., 2011). Three of the 24 did not list integration as a core concept. Two-thirds of the group surveyed listed limits as a core concept; but only 14 thought techniques of integration worthy of mention. There was general consensus that the American first-year calculus curriculum was too crowded, a thought echoed by UQ mathematics academics, yet there was no consensus on which topics should not be studied in first-year.

# Engineering at UQ

UQ has a history of attracting the highest achieving students in Queensland and northern New South Wales. The University as a whole has not had a strong focus on supporting the transition from secondary to tertiary learning environments; however, since 2006 the Faculty of Engineering, Architecture and Information Technology (EAIT) has employed a first-year manager, and implemented a suite of innovations (Crosthwaite and Kavanagh, 2012) to assist students in this transition. Stable retention rates in the range of 87 – 90% in the face of rising student numbers (650 in 2006 to 1000 in 2009) indicate that these innovations have been successful.

The EAIT Faculty includes four engineering schools covering 18 degree specialisations. First-year students study three or four mathematics courses (taught by the mathematics department), depending on whether they did both intermediate and advanced mathematics at high school or just intermediate mathematics respectively. Prior to the mid-1990s, UQ had strict enrolment prerequisites for entry to engineering that included both intermediate and advanced mathematics. In the changes instigated during the mid-1990s, advanced mathematics was one of the secondary school subjects that was no longer required for entry to engineering. Once this prerequisite was removed, the University introduced an advanced mathematics bridging course, predominantly for engineering and science students, to cover the core topics of Queensland's advanced mathematics: functions, differentiation, integration, matrices, vectors, sequences, series, and complex numbers. However, the course runs for 30 hours (one semester), compared with approximately 200 hours at high school. Consequently, it is impossible to teach the same amount of content, and, importantly, students do not have as long a time period to understand and consolidate the material, and develop automaticity and fluency.

The removal of the advanced mathematics prerequisite had a significant effect on the nature of the first-year engineering cohort. In general, only 60 - 70% of first-year engineering students have studied both intermediate and advanced mathematics at school (UQ, 2007-2012). This has left 30 - 40% of students entering engineering not only without an extra 30 hours of more complex integration, but without two years of working and thinking mathematically. These students effectively start on the back foot needing to take the bridging course to get up to the required level of knowledge but also needing to use the concept in engineering science courses.

It is worth noting that UQ does not have a Bachelor of Mathematics, and separate mathematics courses for each discipline (e.g., mathematics, engineering, science, arts) were discontinued in the early 2000s. Since then all mathematics courses contain students from a range of degree programs; however, the majority of students are studying engineering. Consequently, mathematics staff work closely with engineering staff to ensure the content of courses matches the structure of the engineering degree program.

# Diagnostic testing

One of the suite of innovations introduced to address transition issues and the increased diversity of backgrounds, knowledge, and abilities has been the reintroduction of diagnostic testing. From 1972 to 1995, incoming UQ engineering students were given a diagnostic test based on secondary school intermediate and advanced mathematics syllabi (Pemberton and Belward, 1996). In 2007, UQ engineering and mathematics academics reintroduced the investigation into first-year students' abilities via a quiz administered in their first lecture of semester. Students who had studied intermediate and advanced mathematics subjects in secondary school performed better on all questions than those who had just studied intermediate mathematics, and students performed considerably better in topics to which they had more exposure (Jennings, 2008, 2009). Questions on calculus, an area only studied in Years 11 and 12, had the lowest success rate. However, the results suggest that for *both* groups, students' understanding of the topics most recently studied, in this case,

differentiation and integration, appear not to have been strongly consolidated, with students not having developed automaticity and fluency.

From 2009 the quiz became a competency test which included physics and chemistry along with slightly modified mathematics questions. It was offered online to all first-year engineering students before semester began. Analysis of this quiz can be found in Jennings (2009, 2011) and Kavanagh et al. (2009); however, each year the findings are very similar to those from 2007.

As it stands the competency test gives teaching staff some idea of what mathematics the students can do, but it does not give staff much of an idea of what students *understand*. Nevertheless, the quiz has been shown to be very successful in prompting students to revise assumed knowledge that has been poorly remembered, and reducing the knowledge expectation gap between lecturer and student (Kavanagh et al., 2009). This success underpins this current initiative which aims to provide mathematical integration quizzes prior to relevant courses in order to allow students to revise, and academics to more accurately judge cohort knowledge.

In addition to the pre-semester test results, engineering and mathematics lecturers observed that students' differentiation and integration skills were poor not only at the beginning of first semester when students had just begun university, but that they were still poor after two semesters of mathematics (i.e., upon entering second year), and that things did not improve over the four-year degree program. To quote a fourth-year civil engineering lecturer, "My students can't integrate the square root of *x* at the end of fourth year!"

This project therefore has several aims:

- 1. to determine what integration is used across the BE program;
- 2. to determine what lecturers think of students' integration skills; and
- 3. to develop online quizzes for staff to give students to determine their integration ability for courses.

Given the success of the pre-Semester 1 competency test, the integration quizzes will be of similar format, enabling students and staff to receive instantaneous feedback.

### Incoming cohort knowledge (mathematical integration)

Integration is approximately one-sixth (35 hours) of the Queensland Years 11 and 12 intermediate mathematics (Mathematics B) syllabus and covers the following topics:

- MB1. Definition of the definite integral and its relation to the area under a curve.
- MB2. The value of the limit of a sum as a definite integral.
- MB3. Definition of the indefinite integral.
- MB4. Rules of integration:  $\int a f(x) dx$ ,  $\int [f(x) \pm g(x)] dx$ ,  $\int f(ax+b) dx$ .
- MB5. Indefinite integrals of simple polynomial functions, simple exponential functions,
  - $\sin(ax + b)$ ,  $\cos(ax + b)$  and  $1 \div (ax+b)$ .
- MB6. Use of integration to find area.
- MB7. Practical applications of the integral.
- MB8. Trapezoidal rule for the approximation of a value of a definite integral numerically.

### Advanced mathematics (Mathematics C) includes an extra 30 hours of integration, including:

- MC1. Integrals of the form:  $\int [f'(x) \div f(x)] dx$ ,  $\int f(g(x)) \cdot g'(x) dx$ .
- MC2. Simple integration by parts.
- MC3. Development and use of Simpson's rule.
- MC4. Life-related applications of simple, linear, first-order differential equations with constant coefficients.
- MC5. Solution of simple, linear, first-order differential equations with constant coefficients.

The pre-semester competency test results showed that while most students can integrate a polynomial, only two-thirds of the cohort could evaluate an elementary definite integral (the integral from x = 0 to 2 of 2x+3, with respect to x).

# Mathematical integration at UQ

Twenty-three different integration topics are taught in the three (or four if the bridging course is required) university mathematics courses that all engineering students study. They are shown in Table 1.

Table 1: Integration topics taught in mathematics cou	urses taken by BE students at UQ
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Riemann sums	Integration by substitution (2D,3D)
Integration of polynomials	Change in coordinates
Integration of exponential functions	Stokes' theorem/ Divergence theorem
Integration of logarithmic functions	Line integrals
Integration of trigonometric functions	Surface area/volume integrals (2D, 3D)
Integration by substitution (1D including	Moments of inertia/Centre of mass
trigonometric and rational)	Fourier series
Integration by parts	Convolutions
Integration by partial fractions	Lebesgue integration
Improper integrals: fundamental theorem	Fubini/Fatou/Dominated convergence
Volumes of revolution	Contour integrals
Surface area of revolution	Integro-Differential equations

The overlap between advanced mathematics and what is taught in the bridging course is not a perfect match; only MC1 is covered in the bridging course. MC2, 4 and 5 are taught at various times in the other three compulsory mathematics courses. Simpson's rule, MC3, is never covered. Over the last five years students who have studied the bridging course have had considerable difficulty with integration by substitution questions on end of semester examinations. Even after another semester of study (Calculus & Linear Algebra 1) students' results on integration questions (including integration by substitution, parts, and partial fractions) are disappointing, with average marks in all questions less than 50%.

# Methodology

Data on integration topics used in engineering courses across the four-year degree program and across the various disciplines was collected through the use of an online survey, sent to 61 lecturers. The same survey was used to obtain lecturers' perceptions of the level of students' integration skills (poor, satisfactory/average, above average). Space was available for lecturers to make general comments.

Lecturers were also asked if they wanted online competency quizzes, similar to the one undertaken by incoming first-year students, for each integration topic used in their courses. This will support the second phase of the research in which quizzes and appropriate revision exercises will be made available to lecturers to give students at an appropriate time.

# Results

Twenty academics (33%) across the BE program responded to the survey, covering 40 courses from first-year through to fourth-year. All four Schools were represented, with civil having the highest number of courses using integration, followed by mechanical and chemical, then mining and electrical. Five general first-year engineering courses also required integration. The BE requires completion of a total of 32 courses: eight courses per year for four years, so the survey response was adequate. Responses showed that each of the 23 integration topics taught to engineering students (Table 1) was either assumed to

have been taught in a previous course, revised, or taught again. Each of the 23 topics was mentioned in connection with at least three courses.

The main topics nominated by lecturers as those they assumed students have studied and understood in previous courses were the more introductory topics of integration of polynomials (75% of lecturers), trigonometric functions (61%), and exponential functions (55%). These were followed by integration of logarithmic functions (44%) and integration by parts (42%). Of the more complex topics, change of coordinates (29%), surface area/volume integrals (2D, 3D) (28%), moments of inertia/centre of mass (28%), and line integrals (24%) were the main topics listed as assumed knowledge. Some topics were nominated by one discipline only: contour integrals by civil engineering lecturers, and Integro-differential equations, moments of inertia/centre of mass, and Fourier series by mechanical engineering lecturers.

It was interesting to note that of the introductory topics, Reimann sums was the least nominated. By definition, a definite integral is the limit of a Riemann sum, with Reimann sums forming the foundation of all integration at university. For example, when learning multiple integrals used to find volumes under surfaces not just areas under curves, an understanding of Riemann sums is crucial. In addition, not every function has an integral that can be worked out analytically.  $f(x) = e^{Ax^2}$  is one such function. To determine this integral one would generally use a software package to take a Riemann sum over a very large number of segments. Perhaps the engineering lecturers who responded to the survey tend to deal with only functions that can be integrated analytically. However, one lecturer who teaches a first-year core engineering course and a third-year chemical engineering course, and who thought students' integration skills were poor, said:

Students would be expected to be able to numerically integrate any function. I would have hoped that there was no resistance to integration in either of the two courses. For some students in both courses it is clear that there are some difficulties.

This lecturer, however, did not revise Reimann sums in lectures, nor request a Reimann sum online revision quiz.

Half of the lecturers felt that students' knowledge of integration was at best average or satisfactory, whereas the other half felt that the students' performance was below expectations (Table 2). Interestingly, of those who said 'satisfactory', the majority taught courses where not much integration was used.

	Poor	Average/satisfactory	Above average
1 <sup>st</sup> year course	3	0	0
2 <sup>nd</sup> year course	2	4	0
3 <sup>rd</sup> year course	6	5	0
4 <sup>th</sup> year course	4	3	0

Table 2: Lecturers'	perceptions on	students' integ	ration skills a	according to	vear level
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Some lecturers made extra comments, saying that the problem is not just in integration, but mathematical concepts in general. As a third-year mechanical engineering lecturer said:

The problems are with other mathematical concepts, not integration. Some students believe all matrices must be square, for instance. Eigenvalue analysis is another weak area.

And a fourth-year fluid mechanics lecturer, who uses all 23 topics, said:

The engineering students should have a better math background before entering UQ and they should do more math in 1st Year.

A second-year mechanical engineering lecturer said his course involved integration using technology instead of by hand, saying:

a computational-tools-oriented course where we show them how to use the brute force (metaphorically speaking) of the computer to replace analytical finesse the Maths fellows are trying to instill. So, I don't mind the gaps in their integration skills. For the gaps in my integration skills, I use Maxima.

Only one comment, from a third/fourth-year civil engineering lecturer, was surprising: *I don't really know. I just assume they can do it.* This lecturer indicated that the topics of integration of polynomials, exponentials, logarithmic, and trigonometric functions, along with line integrals, and moments of intertia/centre of mass were assumed knowledge and not revised in class. Given the comments from the other third- and fourth-year lecturers it would be interesting to see how students in this lecturer's courses performed on integration questions.

Ten lecturers requested online revision quizzes as shown in Table 3.

 Table 3: Requested online integration quizzes

Торіс	No. of courses	Year level
Integration by parts	7	1,2,3,4
Integration of exponential functions	6	1,2,3,4
Moments of intertia/Centre of mass	6	1,2,3,4
Integration of polynomials	5	2,3,4
Integration of exponential functions	5	1,2,3,4
Integration by partial fractions	4	2,3,4
Integration by substitution (1D: inc. trig. & rational)	4	1,2,4

One lecturer even commented as to the value of revision thus supporting the second stage of this initiative where appropriate resources are provided to bring students up to speed before the beginning of the course:

Once revised, they are usually back up to speed. The coverage is okay from the maths courses, it is more that the students are not retaining it.

# Conclusion

Mathematical integration is a fundamental skill that all engineering students will be required to use at various stages and levels throughout their undergraduate and professional lives. An entry competency test shows that students are not retaining, or perhaps did not understand fully, secondary school knowledge of the concept. Results from a survey at UQ across year levels and engineering disciplines indicated that a significant number of engineering lecturers think students' integration ability and understanding is poor. This is supported by end of semester examination results. Therefore, as integration is required for many courses across all years and disciplines of the degree program, it is important that this deficit be addressed. In order to reduce the knowledge expectation gap and to prompt students to revise relevant integration topics, the next phase of this research will see integration competency tests administered before key courses and appropriate resources developed.

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