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Teaching threshold concepts in engineering mathematics using MathsCasts

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BACKGROUND
Engineering students undertaking mathematical subjects often encounter difficulties with specific concepts taught in their units. This presents a pedagogical challenge as teachers need to provide students with a sound foundation in mathematical concepts. Threshold concepts denote concepts that are essential to knowledge and understanding within particular disciplines as they act like conceptual portals that once crossed enable students to comprehend a topic not previously understood. In turn this enables the learner to progress to higher levels of learning. Grasping a threshold concept transforms student perceptions of the subject area, and they are better able to relate the topic to core ideas in wider fields of study. We note that threshold concepts discussed in the literature are mostly at a broader scale, e.g. numbers and functions, and do not pin point particular micro-concepts within these larger concepts (Worsley, Bulmer and O'Brien,2008; Petterson, 2012)

PURPOSE
In this study, we investigate how threshold concepts can be identified down to the level of individual mathematical examples.

DESIGN/METHOD
We undertake a content analysis of explanations of mathematical problem solving in MathsCasts. MathsCasts are short and focused videos of mathematical explanation that were specifically developed to support first-year engineering students' learning of mathematics.

RESULTS
While all MathsCasts, by design, address conceptually complex knowledge, it is not immediately clear if these are also threshold concepts. Through application of the criteria for threshold concepts to selected MathsCasts, we indeed found two exemplars that appear to fulfil most criteria for threshold concepts.

CONCLUSIONS
We have found this exercise of identifying threshold concepts in MathsCasts much more involved than initially expected, but believe that it is a productive basis for pedagogy that needs further investigation. For instance, students will need to be interviewed on their views to corroborate our findings, and the acquisition of (micro) threshold concepts and cognitive transformation in students will need to be verified.

KEYWORDS
Threshold concepts, engineering mathematics, MathsCasts, troublesome knowledge

Introduction

Engineering students undertaking mathematical subjects often encounter difficulties with specific concepts taught in their units due to the abstract nature of the discipline. However, complex and abstract concepts are an essential part of mathematical understanding and need to be mastered before progressing to the next level and applying new understandings gained to other disciplines. Students encounter complex mathematical concepts across a number of learning contexts. Lack of a strong foundation in key mathematical concepts can be a major hurdle for academic success in their studies. By grasping essential mathematical principles, students are able to see connections, and develop new ways of thinking that are necessary for future learning in their discipline.

Mathematics is a core subject in engineering education. To increase student retention, many Australian universities have established drop-in mathematics support centres, where students can seek help and guidance in learning core concepts (MacGillivray, 2008). At Swinburne University of Technology, the Maths and Stats Help Centre offers help from tutors when needed. In order to extend the support hours from usual business hours to any time support, videos have been produced that can be accessed online. MathsCasts (Loch, Gill and Croft, submitted) are short, targeted online video recordings with narration. They explain a mathematical concept and walk learners step-by-step through the solution to a problem. The topics are chosen specifically from those that students who visit mathematics support centres ask for help on. MathsCasts are an open educational resource and created through international collaboration between Swinburne University, the University of Limerick and Loughborough University. At Swinburne University, the MathsCasts are recorded for first and second year Engineering students studying mathematics units. Students can download these to their smart phones, tablets or access them from a computer, whether they are on or off campus. The effectiveness of screencasts as pedagogical tools and their impact on student learning at university has been investigated in several research studies, see for example (Lee, Pradhan and Dalgarno, 2008).

The design of the MathsCasts is in the form of demonstration and explanation of procedures by an expert teacher, which students watch and listen to. Evaluation of the MathsCasts up to now has relied on student feedback on the value of these MathsCasts to their own learning of the subject (Loch, Croft and Gill, 2012). We have also revised the instructional design approach to screencasts by investigating how self-regulation could be fostered by adopting more active and experiential approaches that encourage students to set goals and engage in transfer tasks following a viewing of the screencasts (McLoughlin and Loch, 2012; Loch and McLoughlin, 2011).

Rationale for the study

The examples recorded in MathsCasts have been selected and recorded by practitioners based on “troublesome” concepts identified from face-to-face interaction with students seeking help in the maths support centre. Since many student visits to the support centre are driven by assessment tasks with imminent due dates, students often ask for help on routine problem solving, with the occasional student wanting to deeply engage with and understand a complex concept. To improve the educational outcomes of students who access MathsCasts, this study introduces an initial investigation into how threshold concepts may be addressed and explained in MathsCasts. While researching the literature on threshold concepts we found no guidance on what a threshold concept may be at individual example level (see next section). In this paper, we therefore describe our first step in this investigation: i.e., our approach to the identification of threshold concepts in selected mathematical explanations down to example level. This preliminary study is a first step to engage in a three stage research process:

i) Stage 1: Analysis of whether MathsCasts now in use address threshold concepts

ii) Stage 2: Revisiting the instructional approach taken in MathsCasts explanations and refocusing on redelivering MathsCasts that teach threshold concepts
iii) Stage 3: Evaluation of whether MathsCasts that focus on teaching threshold concepts are a more effective and engaging pedagogical approach for students.

Outcomes will inform instructional approaches taken for future production of MathsCasts.

Threshold Concepts

What are threshold concepts?

In each discipline there are some concepts that have proven particularly difficult for students to comprehend. These are often labelled “troublesome”. The term *threshold concept* derives from educational theory to denote concepts that are troublesome, but at the same time essential to knowledge and understanding within particular disciplines. According to Meyer and Land (2005; 2003), threshold concepts act like conceptual gateways that once crossed enable students to comprehend a topic not previously understood. The process of learning threshold concepts can either be an immediate or a long term process of comprehension, and is most often scaffolded by a teacher who engages students at a deep conceptual level in their discipline. Once they have been identified, threshold concepts appear to encapsulate and describe the essential learning outcomes for students as they embody a coherent and structured body of ideas that underpin each discipline. Pedagogic research organised around the investigation of threshold concepts offers a fresh way of thinking about students’ learning processes. We will now briefly describe the core features of threshold concepts.

The ‘troublesomeness’ of a concept forms part of the criteria that define it as a threshold concept i.e. students should experience difficulty and discomfort when faced with the concept but they must overcome this as they learn within their discipline. As well as being *troublesome*, we use the following three criteria to define a threshold concept (Meyer and Land, 2003; 2005; Cousin, 2006), and take an approach similar to Boustedt et al. (2007) in identifying how they are addressed.

Threshold concepts are potentially:

- *transformative*, i.e. a significant shift in the perception of a subject takes place once a student begins to understand. In other words, learners now view their subject area in a new way. In certain instances this may also occasion a significant shift in the identity of a learner. This may have a performative aspect, allowing the learner to ‘do’ something they were not able to ‘do’ before.
- *irreversible*, i.e. unlikely to be forgotten, or unlearned only through particular effort. Though this may be difficult to prove, the transformation undergone by the learner is so profound that it is unlikely they will regress to previously held conceptions.
- *integrative*, i.e. understanding of a threshold concept allows the learner to make connections between concepts that were previously ‘hidden’ to them.

Threshold concepts in Mathematics

A review of the literature on threshold concepts in mathematics indicates that while there are many troublesome concepts that students struggle with, identifying threshold concepts among those may not be a trivial exercise. One of the first named threshold concepts is that of *limit* (Meyer and Land 2003). To this were added *fractions, ratios, proportions and percentages* by Long (2009), while Pettersson (2012) justifies the inclusion of the concept of *function*. Worsley, Bulmer and O’Brien (2008), looking at second year university mathematics, suggest *ordinary differential equations*, the *method of substitution*, and *multiple integration*.

Given the hierarchical structure on which mathematics is built, one could even go as far as to suggest that any prerequisite topic is a threshold (a gateway or portal to understanding), troublesome or not, "*opening up a new and previously inaccessible way of thinking about something"* (Meyer and Land, 2006) as new knowledge is formed by extending previously acquired knowledge. The learner cannot proceed until this concept is understood.

In fact, Galligan (2010) proposes personalised threshold concepts, as "*while some concepts may be obviously threshold in mathematics […] others exist at the personal ability level. It may be that any concept at any level of mathematics learning could be threshold to an individual person."*

On the other hand, attempts have been made to explain what is not a threshold concept. A threshold concept is not merely a core idea that is troublesome. Atherton (2011) gives the example of the standard double-entry method in book-keeping which is not a threshold concept, and calls this *ritual knowledge*. Translated into the mathematical context, a method that students learn to solve a mathematical problem, even if they understand why it was developed, is not a threshold concept. While Worsley et al. (2008) suggest three threshold concepts, they also explicitly state that *hyperbolic functions* are troublesome, but not threshold concept in themselves as they are not transformative.

We conclude this section with a reference of interest to engineering education. While not directly focusing on mathematics, the work of Male, Guzzomi and Baillie (2012) investigates threshold concepts that integrate engineering disciplines, with many of these concepts based on mathematical modelling and abstraction.

Method and Results

In this study we are interested in identifying threshold concepts at individual mathematical example level. We undertake a content analysis of existing MathsCasts to identify which, if any, address threshold concepts. While all MathsCasts, by design, address troublesome knowledge, it is not immediately clear if those are also threshold concepts.

While most MathsCasts take students through routine problem solving steps, we were also able to identify MathsCasts that appear to address threshold concepts. We present two of these here, both identified as strong potential candidates:

- The first explains the solution of a particular type of inequality. The commonly taught approach to solve avoids an important mathematical concept: the multiplication by a variable of an unknown sign within an inequality. Student attempts also often avoid a direct approach.
- The second example explains why a commonly observed student solution to finding a derivative of a function raised to the power of another function is incorrect. Often, students do not understand the concept of substitution of all occurrences of a variable inside a function, and the need to use logarithmic differentiation in this case.

Both examples are popular test questions in first year engineering mathematics. From student answers, the marker can easily gauge if the student has any understanding of the relevant major concepts.

MathsCast 1: Multiplication by variable in an inequality

The first example is the inequality shown in Figure 1. While students often still remember how to multiply by a constant of known sign in an inequality and can also solve the respective equation, they do not appear to transfer this prior knowledge to the new concept shown (as laid out in the MathsCast screenshot on the left). This type of question is intentionally solved in a particular way in class (see solution on the right). Students are shown a solution that does not require multiplication by a variable in this inequality, and instead check conditions that need to be fulfilled for a ratio to be positive or negative.

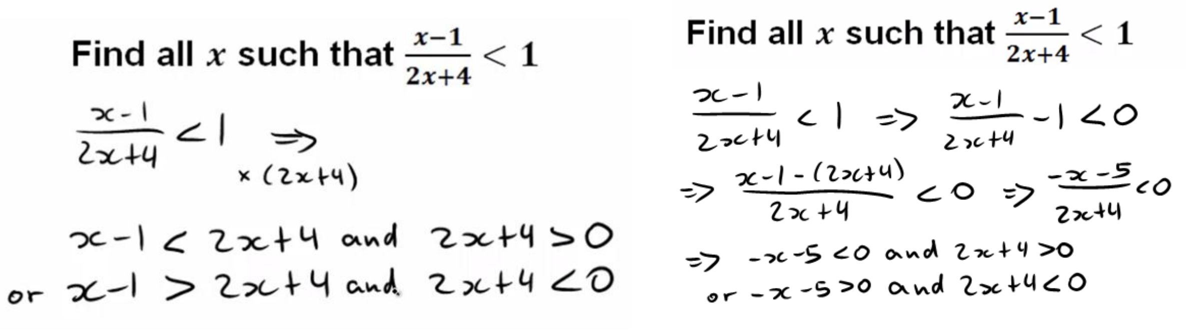

Figure 1: Screenshot of MathsCast "Multiplying by variables in inequalities" (left), and the path suggested to students to avoid multiplication (right).

In fact, a scan through test papers for 34 students attending the same lecture stream in a first year engineering mathematics unit showed that 24 of these solved the question in the way explained in class, although not all found the correct solution. One tried polynomial division of the ratio and gave up. Another unsuccessfully tried a graphical solution. Three students cross-multiplied by the denominator, ignoring the fact that the sign might change (one had transformed the inequality into an equation), but then found the correct solution by trial and error. Four students cross-multiplied as if solving an equation. Of those who did multiply by the denominator across the inequality, none gave a coherent explanation for their answer, with some solutions clearly derived by a method the student had learnt to use but not understood, as they could not explain their working.

This example shows how students are shielded from exposure to a troublesome concept by learning a particular method of solution that avoids multiplication in an inequality. Meyer and Shanahan (2003) found that there are "*implications for the manner in which students are initially introduced to threshold concepts*", one of which is that "*first impressions matter*" and that

> "*efforts to make threshold concepts 'easier' by simplifying their initial expression and application may, in fact, set students onto a path of 'ritualised' knowledge that actually creates a barrier that results in some students being prevented from crossing the 'threshold' of a concept*."

Meyer and Land (2005) comment that simplified explanations to scaffold student learning may act as a false proxy for the threshold concept, "*leading students to settle for the naive version, and entering into a form of ritualised learning or mimicry*", rather than aiding student understanding. Atherton (2011) speaks of a disservice to students when the issue is "*fudged*".

MathsCast 1 was created to directly address simplification by relating the solution to this problem to prior knowledge.

MathsCast 2: Logarithmic differentiation

The second example is shown in Figure 2. The screenshot from the MathsCast "What's wrong? Differentiation with a variable in the exponent" on the left pictures a reproduced incorrect solution similar to that written by a student in a test paper in engineering mathematics. In this solution, the student identifies composition of functions ($u = 2x + 5$ and $y = u^{3x}$), however does not substitude every occurrence of x. The student then takes the x as a constant in the exponent when using the chain rule and power rule to find the derivative. The explanation on the right in Figure 2 is correct, applying logarithmic differentiation to re-write the function in a way that common derivative rules apply. Showing what is wrong with a particular solution to a problem gives the opportunity of explaining misconceptions.

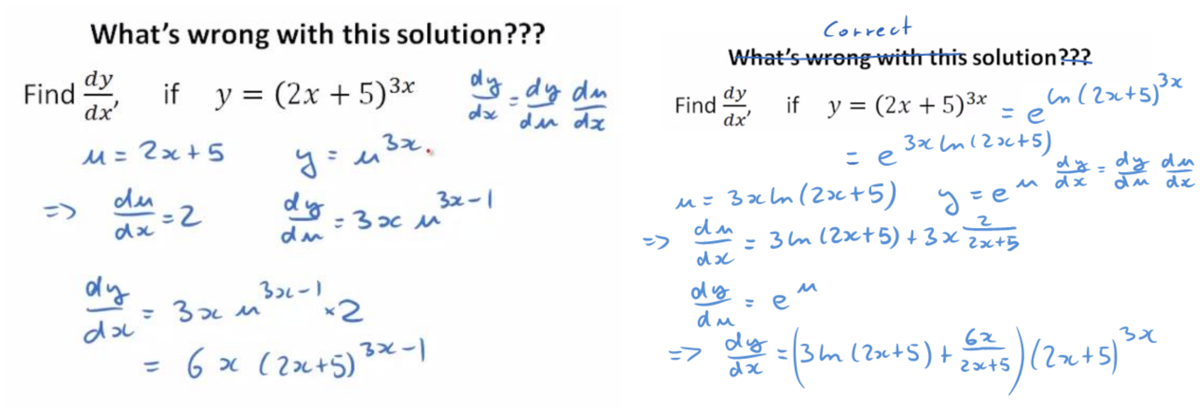

Figure 2: Screenshot of MathsCast "What's wrong? Differentiation with a variable in the exponent" showing an incorrect student solution (left), and the correct solution (right).

MathsCasts that were identified as not addressing threshold concepts usually were not transformative or integrative. Table 1 shows in how far MathsCast 1 and MathsCast 2 address the criteria and are threshold concepts:

Table 1: Threshold Concept Criteria fulfilled by MathsCast 1 and MathsCast 2

Criterion	MathsCast 1	MathsCast 2
troublesome	Identified by the lecturers as troublesome to students, hence taking an approach to avoid direct confrontation with the difficult concept. Solving an equation requires simply transformation, while solving the inequality adds conditions to the solution. This may be counter-intuitive.	Frequently answered incorrectly by students in tests. Basic rules that the student is more familiar with (e.g. power rule) are applied to the wrong context.
transformative	It alters the way that students think about mathematics: It reassures students that mathematics is still about extending previous knowledge rather than reaching a barrier that has to be circumnavigated.	A significant shift in the perception is probably not to be expected here. This MathsCast may have a performative aspect, allowing the learner to 'do' something they were not able to 'do' before.
irreversible	Once understood, solving of inequalities is difficult for the student to unlearn.	Once understood, logarithmic differentiation is difficult for the student to unlearn.
integrative	Tying together two previously known concepts in a new way.	Tying together previously known concepts in a new way.

Conclusions

Before starting this investigation, it was not clear if it was possible to pin point threshold concepts at example level as the literature on this topic is vague. We have indeed identified key ideas in two MathsCasts that appear to fulfil most criteria for threshold concepts. This finding is significant as the aim of this study is to initially highlight threshold concepts, and then to eventually inform instructional approaches taken for future production of MathsCasts. Our plan is to identify how we can focus MathsCasts better, by not just explaining how to solve particular problems, but getting students to understand concepts that are barriers to learning.

We have found this exercise of identifying threshold concepts in MathsCasts much more involved than initially expected, as the point where a troublesome concept becomes a threshold concept when looking at individual examples and small concepts is not easily determined. This is where the literature is lacking, as threshold concepts discussed are mostly at a broader scale, e.g. numbers and functions, and do not pin point particular micro-concepts within these larger concepts. Some attempt was made by Worsley et al. (2007) to give more specific directions. We suggest that it may be reasonable to talk about micro-threshold concepts in the MathsCasts context and we intend to investigate this idea further in the near future.

While we are certain that the two MathsCasts selected are troublesome for students by looking at their attempts of solving these problems, students will need to be interviewed on their views to corroborate our findings. Similar to the study undertaken by Boustedt et al. (2007), it may also be useful to discuss difficult concepts with mathematics lecturers in order to establish if they are threshold concepts

Following our realisation that many MathsCasts are covering explanations of routine problem solving, we will rethink our selection and types of examples to be covered. For instance, future MathsCasts may be specifically developed as a starting point to prepare students in selected foundation level concepts that would enable them to progress to better comprehension of deeper levels of complexity in the subject. Future research should lead to several other areas of investigation such as:

- How can teachers recognise the acquisition of threshold concepts in students?
- Is self-reporting by students sufficient?
- Where and how can teachers find evidence of conceptual transformation and how can they scaffold students to better recognise and manage their understanding of troublesome concepts?

These remain some of the enduring questions that underpin our research and will be part of our future research into the creation of effective MathsCasts for engineering students that embed threshold concepts.

References

Atherton, J. (2011). Doceo; What are not threshold concepts [On-line: UK] Retrieved 26 August 2012 from http://www.doceo.co.uk/tools/thresholds_7.htm

Boustedt, J., Eckerdal, A., McCartney, R., Mostroem, J., Ratcliffe, M., Sanders, K. and Zander, C. (2007). Threshold concepts in computer science: Do they exist and are they useful? *SIGCSE'07*, March 7–10, 2007, Covington, Kentucky, USA, p.504-508

Cousin, G. (2006) 'An introduction to threshold concepts', *Planet,* 17, pp. 4-5.

Galligan, L. (2010). Examining personal and contextual threshold concepts in academic numeracy. Presentation at 3rd Biennial Threshold Concepts Symposium, Sydney, 1-2 July 2010. Retrieved 26 August 2012 from http://www.thresholdconcepts2010.unsw.edu.au/Abstracts/GalliganL.pdf

Galligan, L., Wandel, A. and Hartle, T. (2011) Scaffolding distance learning in mathematics for engineering: identifying key troublesome knowledge. In: 1st International Conference of STEM in Education, 26-27 Nov 2010, Brisbane, Australia.

Lee, M. J. W., Pradhan, S. and Dalgarno, B. (2008). Using screencasting to scaffold exercises and promote cognitive engagement for novice object-oriented programmers. *Journal of Information Technology Education, 7*, 61–80.

Loch, B. and McLoughlin, C. (2011). An instructional design model for screencasting: Engaging students in self-regulated learning. In G.Williams, N. Brown, M. Pittard, B. Cleland (Eds.) Changing Demands, *Changing Directions. Proceedings ascilite Hobart 2011.*

Loch, B., Croft, A. and Gill, O. (2012). MathsCasts – enhancing the resource base for mathematics support centres. In: Media enhanced teaching and learning: case studies and evidence of effective use / Peter Rowlett (ed.), pp. 22-23, Mathematical Sciences HE Curriculum Innovation Project, UK.

Loch, B., Gill, O. and Croft, A. (submitted). Complementing mathematics support with online MathsCasts.

MacGillivray, H. (2008) Learning Support in Mathematics and Statistics in Australian Universities: A Guide for the University Sector. Australian Learning and Teaching Council, Sydney, 2008.

Male, S., Guzzomi, A. and Baillie, C. (2012). Interdisciplinary threshold concepts in engineering. In Brown, N., Jones S. M. and Adam, A. (Eds.) *Research and Development in Higher Education: Connections in Higher Education*, 35 (pp 151 – 159). Hobart, Australia, 2 – 5 July 2012.

McLoughlin, C. and Loch, B. (2012). Engaging students in cognitive and metacognitive processes using screencasts In Amiel T, & Wilson, B (eds) *Proceeding of Ed-Media 2012, World Conference on Educational Multimedia and Hypermedia.*

Meyer, J. and Land, R. (2003). Threshold Concepts and Troublesome Knowledge: Linkages to Ways of Thinking and Practising within the Disciplines Edinburgh; *Enhancing Teaching-Learning Environments in Undergraduate Courses Project, Occasional Paper 4 [on-line, UK]* Retrieved 26 August 2012 from http://www.tla.ed.ac.uk/etl/docs/ETLreport4.pdf

Meyer, J. and Land, R. (2005). Threshold concepts and troublesome knowledge (2): Epistemological considerations and a conceptual framework for teaching and learning. *Higher Education,* 49: 373–388.

Meyer, J. and Shanahan, M. (2003). The troublesome nature of a threshold concept in economics. Paper presented to the *10th Conference of the European Association for Research on Learning and Instruction (EARLI)*, Padova, Italy, August 26–30.

Meyer, J., Land, R., and Davies, P. (2006) Threshold concepts and troublesome knowledge (4):

Pettersson, K. (2012). The Threshold Concept of a Function – A Case Study of a Student's Development of Her Understanding. Retrieved 26 August 2012 from http://www.mai.liu.se/SMDF/madif8/Pettersson.pdf

Scheja, M. and Pettersson, K. (2010). Transformation and contextualisation: conceptualising students' conceptual understandings of threshold concepts in calculus Higher *Education* (2010) 59:221–241

Worsley, S., Bulmer, M. and O'Brien, M. (2008). Threshold concepts and troublesome knowledge in a second-level mathematics course. In A. Hugman & K. Placing (Eds.) *Symposium Proceedings: Visualisation and Concept Development,* (pp. 139–144). UniServe Science: The University of Sydney.

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