Full Paper

Introduction

This paper extends the work reported on in the AAEE Conference held in Wellington, New Zealand in 2014.

Many students entering tertiary education in the technical fields have poor mathematics abilities (see ‘diagnostic test’ below). This problem is particularly severe in the polytechnic sector. This paper looks at the approach taken at the Manukau Institute of Technology to mitigate this problem.

The poor mathematics ability of the students affects not only their ability to solve numerical problems but also affects their ability to learn technical material (Soderstrom, & Bjork, 2014). In higher level subjects, mathematics is used as a language to explain cognitively complex topics and therefore students need to be fluent in mathematics in order to understand these explanations. If a student is not fluent in mathematics they will be forced to use their working memory to figure out the mathematics that is being used to explain the complex topic rather than using their working memory to comprehend the topic itself. This is particularly a problem because working memory is very limited, i.e. typically an average person can hold only seven independent concepts in working memory at a time (Baddeley, 2004). Therefore if the students have to think about the mathematics being used to do the explaining they will be unlikely to have sufficient working memory to also think about the complex topic being explained. They will then have difficulty understanding and comprehending the new topic, i.e. new learning will fail (Brown, Roediger III, & McDaniel, 2014).

However, if the students are fluent in mathematics, (that is, they do not have to think about the mathematics they are using), they will be able to use all their limited working memory to think about the new topic being explained which in turn will improve the possibility of learning taking place (Willingham, 2009).

This effect of the lack of fluency in mathematics affecting learning applies in particular to the learning of more advanced mathematics (Barclay, Bransford, Franks, McCarrel, & Nitsch, 1974). If students are not fluent in the basic mathematical procedures, theorems, and axioms they will have great difficulty in advancing onto more complex topics for the same reason as described above: their limited working memory will be used in figuring out the basic mathematics rather than the advanced topics when they are being taught the advanced topics. They will then not develop a deep understanding of the higher mathematical concepts, i.e. they will not easily experience the ‘aha’ moments that are required when learning and ultimately understanding higher mathematical concepts. This is because these ‘aha’ moments (or moments of understanding) depend on being fluent in the basics of mathematics (Cumming & Elkins, 1999), (Alexander, Kulikowich, & Schulze, 1994). In addition, the more fluent the students are in mathematics the more likely it is that they will be able to see and understand how the different parts of mathematics interlink.

Finally, one of the important aspects of a tertiary education is developing the ability to undertake self-learning once one has graduated. Because the language of science, technology, and engineering is mathematics it is imperative that students graduating in these fields have a wide and fluent knowledge of mathematics (Bahrick & Hall, 1991), (Ellis, Semb, & Cole, 1998).

The next section describes the study undertaken at the Manukau Institute of Technology to measure the degree of the problem, i.e. the students' poor mathematics ability, and to develop strategies to overcome the problem.

The Background to the Manukau Institute of Technology Study

The study at the Manukau Institute of Technology involved the students enrolling for the three year bachelor of engineering technology degree in electrical and mechanical engineering. The entry requirement in mathematics for enrolling on these programs is year thirteen mathematics with calculus or equivalent. All students entering the bachelor of engineering technology degree were assessed by the students' admission staff to make sure that they met these entry requirements.

At the beginning of the semester the students enrolled in the first year mathematics course (141.514 Engineering Mathematics) are given a diagnostic test. This test uses the school year eleven mathematics syllabus to create the questions. The year eleven syllabus is used based on the hypothesis that year thirteen students, i.e. the students entering the first semester mathematics course, should be able to easily complete year eleven problems. No marks were allocated to the diagnostic test. It was merely explained to the students that the diagnostic test was used to aid the lecturer to target the semester's lectures at the correct cognitive level. Sample problems are given in Appendix 2.

The detailed results of these tests are shown in Appendix 1. It is clear from these tests that the students' mathematical ability is poor. The average mark in the diagnostic test is 40.2% with a standard deviation of 25.0%. Of the 43 students that wrote the test only 15 (34.9%) achieved above 50%: which is usually taken as a pass mark. Only 8 (18.6%) students achieved above 67%; that is, less than 20% of the students could be regarded as being fluent in mathematics (i.e. they knew twice as much as they did not know).

The diagnostic test provided a 'snapshot' of the students' ability in the first week of the semester. It was not possible, from this test, to determine what the reasons were for the students' poor performance. Within the department there is much speculation about the reasons for poor mathematics performance but none of this speculation is evidence based and will not be dealt with further in this paper.

In order to get an indication of how well the students' perception of their mathematical ability corresponded to their actual mathematical ability the students were asked to estimate the mark they thought they were going to obtain in the diagnostic test. The details of these results are also shown in Appendix 1. What these data showed is that not only was the students' mathematical ability poor but they did not realise it was poor. The absolute difference between what the students thought they were going to achieve and what they actually achieved is 12.4%, i.e. 0.5 standard deviations. In addition, as Appendix 1 shows, most of the students over estimated their mathematical ability. This combination of a poor ability in mathematics together with an inaccurate perception of their ability in mathematics makes the problem of students enrolling in engineering degrees particularly egregious. This is because the students do not realise that they have a problem that is going to limit their chances of success in their degree studies (Atir, Rosenzweig, & Dunning, 2015).

The Approach used at the Manukau Institute of Technology to Overcome the Problem of poor Mathematics Ability.

In order to improve the mathematical ability of the students and to make their mathematical ability more fluent two principles of learning were implemented viz. extensive practice and feedback (Ericsson, Kampe, & Tesch-Romer, 1993), (Kang, McDermott, & Roediger, 2007).

To give the students extensive practice in solving mathematical problems all the students were enrolled on MyMathLab Global an online mathematics package published by Pearsons. This package was set up so that each week the students had to complete a quiz consisting of number of exercise/tutorial problems related to the topic covered in lectures during that week. In total 11 quizzes were carried out during the 14 week, one semester mathematics course. In order to encourage the students to do the quizzes, the quizzes were allocated a total of 15% of the students final mark (most quizzes were allocated 1% and some were allocated 2% to give a total of 15%).

An important aspect of any form of learning is feedback on how one's learning is progressing. The MyMathLab Global package has a number of useful online feedback facilities. Firstly, when the students have completed a quiz they get immediate feedback on whether their answers were correct or not. Secondly, while they are doing the quiz there is a 'Help Me' function which allows the students to work through a step-by-step solution of a similar problem to the problem that they are working on. Thirdly, the package has a facility whereby the students can be referred to the section in the e-book that relates to the problem that they are working on. An important aspect of all this feedback is that it is stressed to the students that wrong answers are not a bad thing. Instead it is stressed that wrong answers facilitate learning on condition that the students make sure that they understand why the answer was wrong and how to obtain the correct answer. Fourthly, while the students are working on the quizzes a human tutor is available for questions and feedback.

Each week two hours of formal tutorial time is allocated to doing the quizzes and 4 hours is allocated to traditional lecture classes during which the topic theory and some worked examples are covered.

Analysis of the Results

Appendix 1 shows the detailed results and the raw data used in this study.

At the end of the semester all the students sat a two hour mathematics exam. This exam was more difficult than the diagnostic test because it covered topics learnt during the semester. In particular it included complex numbers, matrices, differentiation, integration, and differential equations, none of which were in the diagnostic test. Appendix 1 shows the results of the diagnostic test and of the exam. It is clear from these results that the exam marks are considerably better than the diagnostic test marks even although the diagnostic test was easier.

In order to formalise this improvement the following was done. Firstly, a t-test was carried out to confirm that the averages of the diagnostic test and the exam were statistically different. As Appendix 1 shows, the probability that the averages were different is 99.9997%, i.e. the exam average was definitely statistically different to the diagnostic test average.

Secondly, the effect size of this difference in averages was calculated and found to be 0.70 standard deviations. In the educational field an effect size of greater than 0.4 standard deviations is regarded as good, i.e. it shows that significant learning has taken place (Hattie, 2009). Therefore an effect size of 0.70 shows that the above approach to teaching mathematics has been very effective.

The main aim of the above study was to improve the students' fluency in mathematics. Using the exam results as a proxy for how fluent the students had become in mathematics it may be hypothesised, with some confidence, that the students are significantly more fluent at the end of the semester than they were at the beginning.

Further Refinements to the Teaching Approach

In future semesters an number of additional teaching techniques are going to be included in the teaching of the engineering mathematics course.

Firstly, instead of a quiz consisting only of questions from a particular topic in the syllabus question from topics covered previously will also be interleaved with the current questions. This idea of interleaving previously covered material in the current material has been shown to have educational benefits; in particular it has been shown to improve the depth of the students' learning and understanding (Schacter, 2002).

Secondly, more emphasis is going to be placed on derivations because it is hypothesized that derivations increase the probability that students will be able see how the different mathematical topics link together and give the students a deeper understanding of what mathematical techniques are applicable in what applications.

Thirdly, short talks on educational theory are going to be incorporated into the 2 hour tutorial sessions. The purpose of these talks is to teach the students how to learn, i.e. to cover learning techniques that have been shown to be effective and to point out learning techniques that have been shown to be ineffective.

Discussion and Limitations

There are a number of limitations and weakness with the above study. Firstly, the sample size is relatively small with 43 students. To overcome this problem the approach will be continued over a number of semesters in order to increase the sample size.

Secondly, no control group was used in the study. This was due to the Manukau Institute of Technology's ethical requirements viz. students cannot be taught differently. All students have to be taught using the most effective teaching methods.

Thirdly, this study has not been independently verified. Ideally a different polytechnic with different staff should repeat this study to see if they obtain similar results. (The author will be more than happy to co-operate with and assist any polytechnic that would like to do the replication.)

Fourthly, the main purpose of the study was to improve the students' fluency with mathematics in order to, inter alia, facilitate their learning of more advanced subjects. This aim is difficult to quantify. In the future, once the current first year students have reached final year, it is planned to survey the academic staff to see if they have perceived an improvement in the students' fluency with mathematics. This is probably the best that may be done but the results of a survey of this type will be only anecdotal with all the problems associated with anecdotal 'evidence'.

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Appendix 1

The following are the diagnostic, examination results, and the students' estimated mark:

Student No.	**Diag Test**	**Exam**	**Student's mark estimate**	**|Estimate –diag. test|**	**ES**
530	6.4	43.8			1.32
337	60.3	37			-0.82
975	69.2	95			0.91
80	94.9	87.1			-0.28
120	75.6	91.7			0.57
514	47.4	76.3			1.02
752	21.1	47.5			0.93
177	35.5	71.3			1.26
610	48.7	83.8			1.24
528	56.6	80.9			0.86
252	81.6	55			-0.94
962	15.8	36.5			0.73
449	3.9	7.1			0.11
569	38.2	90			1.83
964	5.3	40			1.22
125	72.4	69.6			-0.10
906	11.8	39.3			0.97
539	89.5	94.2			0.17
270	38.2	87.5			1.74
158	30.3	30			-0.01
347	13.2	59.2			1.62
607	35.5	59.4			0.84
995	24.4	31.1	50	25.6	0.24
861	53.9	41.9	70	16.1	-0.42
962	46.2	50.4	65	18.8	0.15
217	16.7	23	20	3.3	0.22
906	23.1	90	25	1.9	2.36
577	35.9	90	20	15.9	1.91
447	71.8	87.5	80	8.2	0.55
394	34.6	87.9	63	28.4	1.88
339	50	50.8	40	10	0.03
963	53.9	66.3	50	3.9	0.44
85	34.6	48.1	55	20.4	0.48
125	60.3	90	70	9.7	1.05
580	39.7	100	70	30.3	2.13
893	16.7	27.5	30	13.3	0.38
877	23.1	94.4	20	3.1	2.52
517	25.6	5.8	20	5.6	-0.70

98	2.6	9.2	18	15.4	0.23
682	21.8	21.9	30	8.2	0.00
779	62.8	64.1	60	2.8	0.05
269	5.1	32.1	20	14.9	0.95
627	74.4	80	70	4.4	0.20

The following is the statistical analysis of the above results:

	Diagnostic Test	**Examination**
Assessment average	40.2	59.9
Assesssment std. dev.	25.0	27.8
Maximum Mark	94.9	100.0
Minimum Mark	2.6	5.8
Median	35.9	59.4
Combined std. dev.	28.1	
Overall effect size	0.70	
Number of students	43	
95% tolerance on mean	7.86	8.75
Upper/Lower 95% limit	48.1	51.1
Student-t Test	0.000003	
Difference between test and estimate	12.4	
Maximum effect size	2.52	
Note: all values are in %		

Appendix 2

The following is a selection from the questions used in the diagnostic test:

1. Remove the brackets: $-3(5x-2y)$
2. Evaluate: $\left(\frac{1}{3}\div\frac{1}{6}\right)+\frac{1}{2}$
3. Simplify: $\left(\frac{x}{3}\right)^3 x^3$
4. Factorise: $x^2+11x+28$
5. Solve the equation: $7x-16=\frac{2}{3}x+4$
6. Draw the graph of: $y=x^2-3$
7. Find the distance between the points (2, –3) and (5, –1).
8. Calculate the value for $\log_9 9$
9. Write the following as the logarithm of a single number: $\log_4 7+\log_4 5$

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