A double-edged sword: Use of computer algebra systems in first-year Engineering Mathematics and Mechanics courses

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Introduction

Many secondary-level mathematics students have experience with graphical calculators from high school. For the purposes of this paper we define graphical calculators as those able to perform rudimentary *symbolic* manipulation and solve complicated equations requiring very modest user knowledge. The use of more advanced computer algebra systems e.g. Maple, Mathematica, Mathcad, Matlab/MuPad is becoming more prevalent in tertiary-level courses. This paper explores our students' experience using one such system (MuPad) in first-year tertiary Engineering Mathematics and Mechanics courses.

The effectiveness of graphical calculators and computer algebra systems in mathematical pedagogy has been investigated by a multitude of educational researchers (e.g. Ravaglia et al. 1998). Most of these studies found very small or no correlation between student use of graphical calculators or exposure to computer algebra systems with future achievement in mathematics courses (Buteau et al. 2010).

In this paper we focus instead on students' attitude towards a more advanced standalone computer algebra system (MuPad), and whether students' inclination to use the system is indicative of their mathematical understanding.

Method

A pre-use survey (Appendix 1) was distributed to the students to gauge their previous experience with using graphical calculators and familiarity with computer algebra systems. The class was gradually introduced to MuPad via fifteen-minute tutorial demonstrations in a weekly computer laboratory. A written assignment worth 10% of their final grade was made available in the fifth week of the semester. Students had approximately nine weeks to complete the assignment. A post-use survey (Appendix 2) was distributed at the end of the semester to determine if student attitudes and intended future use of MuPad had changed.

As two of the questions in the post-use survey refer directly to the written MuPad assignment a few further details about that assignment are provided. The assignment consisted of four sections:

1. Complex Numbers
2. Algebra/Derivatives
3. Integration
4. Matrices

The first three sections were weighted 20% each and the Matrices section was worth 40% of the total assignment. Each section contained three questions of equal mark value and the student was instructed to attempt only ONE question from each section. Every question was also 'individualised' by changing the question values to be based on two digits from the student's ID number. In this way, no student could merely copy results from another classmate.

Student answers were further analysed to determine if use of MuPad had enhanced their understanding of the course material. This was achieved by a line-by-line reading of their answer code and comments.

A scatterplot of the students' final exam results against their MuPad assignment mark was also produced to determine whether Rodriguez's (2019) conclusion that use of computer algebra systems had no effect on student exam performance was upheld.

Results

From the pre-use survey, only one-quarter of the class had owned and used a GC (graphical calculator) in the past. Those students did not know whether their GC could do symbolic manipulation and were divided equally on whether they intended to use a GC in this or any other course during the semester. The free response question 7 generated some interesting comments which reflect the students' varying background and attitude to mathematics.

Given a set of parameters a symbolic manipulation package can find critical points pertaining to the graphical form of the equation input into the program.

Can solve any algebra expression or question that has constant numbers and different variables.

Solve systems of equations by balancing equations, substituting values and using mathematical rules.

A program capable of doing algebra, plotting graphs and doing matrices.

A coding package that can be used to solve, or plot equations. This is helpful when you need to do a large variety of calculations using the same codes.

Turning now to the post-use survey, the results were divided between those students having GC experience and those without. Unfortunately, survey response rates and the class size were not large enough to perform any meaningful statistical analysis, so the emphasis will instead be on analysis of the responding students' individual MuPad assignment answers.

Students with GC experience (one quarter of respondents)

These students spent from 4—10 hours doing their MuPad assignment. One student reported using the package frequently in other homework or assignments whilst the others said they never did. There was no common topic found most challenging and all students reported that using MuPad had helped their understanding of the course at least somewhat. All students said they would also use the package in further courses (not Mathematics) involving calculation.

Students with no GC experience (all other respondents)

There was great variability in time spent on the assignment with two of the students reporting less than two hours, two more than 10 hours and the remainder 4—6 hours. Responses to the other questions followed a similar pattern to the other group, with no letter choice a) to d) clearly standing out. Most reported that use of MuPad had helped in their understanding of the course material and none said they would never use it in any future (non-Mathematics) course.

Free-response comments on MuPad

Only one of the students with GC experience made further comment on MuPad whilst nearly all of the students with no GC experience did. A selection of their comments follow;

I found MuPad to be relatively simple and effective in improving my understanding of Complex numbers, Differentiation and Integration however I found Matrices far more difficult and got no benefit from attempting to solve the problem, as my struggles came from trying to understand the operation of the software rather than trying to understand the Matrices problem itself.

Difficult to navigate/learn the correct commands without guidance. Useful to check answers to difficult equations/problems when you are confident in how to input commands.

Great tool that can be used well as an engineering student. Would have liked to have a short course in MuPad only before having to use it for Maths course.

MuPad is a good way to solve a problem that calculator can't solve. It's a very good app to use in the future.

The MuPad assignment mark and the final exam mark are presented in the scatterplot (Fig 1). Analysis of the plot shows no trend or corresponding regression and the r-square value is 0.04.

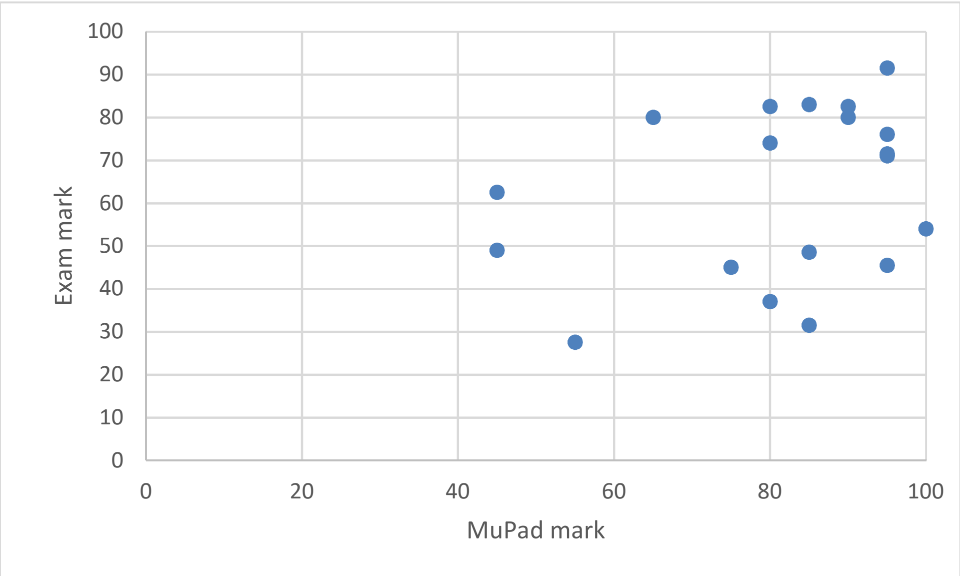

Figure 1: Scatterplot of exam mark against MuPad assignment mark

Analysis of Assignment Answers

To preserve student anonymity, only grouped comments on the trends evident in the assignment answers are presented. Despite our attempt at 'individualising' the questions, it was clear that the class had decided amongst themselves which the 'easier' questions were in each of the four sections, as the same questions were attempted by a large proportion of the class.

Complex Numbers

In this section the question most attempted was an evaluation of the expression

$$T = \frac{20}{15 + jw}$$

for w = 1A, 10B and 2AB rad/s where A and B were the student's last two digits of their student ID number and $j = \sqrt{-1}$. The only major error was in entering the expression into MuPad as $20/15 + jw$, forgetting that as with all calculators or packages, BEDMAS applies. The only other question attempted was use of the solve command to find solutions to the equation $z^3 = A + Bj$. The most common error was to misinterpret the MuPad solution as consisting of four solutions because the package uses placeholders for complicated expressions and displayed the three solutions as combinations of those four placeholders.

Differentiation

The most popular question was to find the velocity and the acceleration for a moving object with displacement

$$s(t) = 5e^{-2t}\cos(3.At + B))$$

(Note this is a decimal point between the 3 and A, not a symbol for multiplication).

This was possibly because the students should be able to perform the required differentiation by pen and paper, although of course the reverse was intended! The main observation here was that weaker students did not use MuPad to store (assign) the dependent variable, instead copying the result for velocity into the differentiation command to obtain velocity. Some students also did not recognise that 'e' has no special meaning in MuPad (it uses exp instead to stand for the exponential function), so that the derivative of e^{-2t} was evaluated as $-2e^{-2t}\ln e$.

Integration

Again, the most popular choice of question appeared to be driven by appearance of simplicity, which was to evaluate the definite integral given below.

'The pressure of a gas in a cylinder is given by $P = \frac{nRT}{V}$ and the work done by the piston to compress the gas is given by $W = \int_{V_1}^{V_2} P\, dV$.

Given that: $n = 0.2A$ mole (number of moles of gas)

$R = 8.314\ \frac{\text{m}^3\text{Pa}}{\text{mol K}}$ (gas constant, K is kelvin = °C + 273)

$T = 2B00\ °\text{C}$ (This needs to be in Kelvin.)

Find the work done by the gas on the piston as the gas expands from 0.01 m^3 to 0.09 m^3.'

A common error was to change the limits of integration to $e^{0.01}$ and $e^{0.09}$ which perhaps indicates those students were thinking about the integral themselves rather than 'trusting' MuPad to perform the integration. Those students 'knew' the integral was $\ln(V)$ and then erroneously concluded that to 'undo' the natural logarithm involved using the exponential.

Matrices

As noted previously the questions in this section were worth double the other sections reflecting the extra understanding of MuPad required and work involved to input the matrices.

The most popular question of the three was to solve

$$-vy_1 + y_2 = -u$$
$$y_1 - vy_2 + y_3 = -4u$$
$$y_2 - vy_3 + y_4 = -9u$$
$$2y_3 - vy_4 = -16u$$

where u and v were parameters dependent on the student's ID number (A and B as before). The majority of students were able to use MuPad to create the matrices needed to solve the equations. Finding the inverse of the coefficient matrix was the only method students used which was unusual because elimination and Cramer's rule were covered in the course material. When students were asked to do an easier problem in the exam, most chose elimination or Cramer's rule.

Discussion and Recommendations

It was expected that students using graphical calculators in the past would adapt more readily and be more enthusiastic about using MuPad. However, the post-use survey results indicated this was not the case. A possible reason was that most of the class struggled initially with the syntax and input instructions, which are completely different from the menu-driven systems those students with GC experience may have been familiar with.

Whilst the use of computer algebra systems allows more 'real-world' engineering examples to be tackled, it is unwise to introduce them too early in a mathematics course. The focus of the mathematics course should be on understanding fundamental concepts and techniques. This opposes the recommendation provided by educators at the secondary level (e.g. Heid, 2002) who propose resequencing the syllabus to take advantage of the power of computer algebra systems. Some evidence that students prefer this sequence were borne out by the overwhelming preference for the 'easy' questions in each of the four sections, when proper use of MuPad should render the 'complexity' of the question a minor consideration. However, this may again be due to difficulty in mastering the syntax.

In line with the findings of Rodriguez (2019), our results in Figure 1 show no correlation between student use of a computer algebra system and overall achievement in student performance. We did not measure student satisfaction or motivation with a quantitative scale as in that study, but our survey results support the findings that those attributes were improved. This is supported by both the higher response rate of the non-GC group to question 7 and the generally positive response of all students to question 4 in the post-use survey.

In future work, the pre- and post-survey will be administered again in both first-year Mathematics and Mechanics courses with the aim of obtaining enough responses to perform meaningful statistical analysis. Focussed in-depth interviews with student volunteers will also be conducted to investigate their reluctance to use the package in tackling 'real-world' problems. The authors would also like to extend an invitation to other institutions using computer algebra systems to collaborate in further research.

References

Buteau, C., Marshall, N., Jarvis, D., Lavicza, Z. (2010) Integrating Computer Algebra Systems in Post-Secondary Mathematics Education: Preliminary Results of a Literature Review. *International Journal for Technology in Mathematics Education* 17(2), 57—68.

Heid, M.K. (2002) How theories about the learning and knowing of mathematics can inform the use of CAS in school mathematics: One perspective. *International Journal for Technology in Mathematics Education* 9(2), 95—112.

Ravaglia, R., Alper, T., Rozenfeld, M., Suppes, P. (1998) Successful pedagogical applications of symbolic computation. In: Kajler N. (eds) Computer-Human Interaction in Symbolic Computation. Texts and Monographs in Symbolic Computation (A Series of the Research Institute for Symbolic Computation, Johannes Kepler University, Linz, Austria, pp. 66-81). Springer, Vienna

Rodriguez, M. (2019) Impact of Implementing Graphing Calculators on College Algebra Students' Performance, Satisfaction, and Motivation. *International Journal of Learning, Teaching and Educational Research* 18(6), 96—109.

Appendix 1

The pre-use survey consisted of the following seven questions

1) What experience do you have with graphical calculators (GC)?
    a) What's a graphical calculator?
    b) Heard of them but never used one
    c) Owned one, used sometimes
    d) Owned one, used often
    e) Couldn't have succeeded in previous course without one.

If you answered either a) or b) to Question 1 you can skip to the **last question** in this survey

2) Could your GC do symbolic manipulation (algebra)?
    a) Yes
    b) No
    c) Don't know
    d) Uncertain what this question means

3) If you answered a) in the previous question, did you actually use your GC to help with algebra in your previous study?
    a) Always
    b) Sometimes, depending on the question
    c) Never
    d) Couldn't figure out how to use this feature.

4) Are you intending to use a GC for this Maths course? Y or N

5) Are you intending to use a GC for any of your other courses this year? Y or N

6) Are you aware of any standalone symbolic manipulation packages (e.g. Mathcad, Maple, Matlab)? Y or N

7) In your own words, write a sentence or two describing what you think a symbolic manipulation package can do.

Appendix 2

The post-use survey contained these six questions

1) How long in total (hours) did you spend on the MuPad assignment?
    a) 0—2
    b) 2—4
    c) 4—6
    d) 6-10
    e) More than 10

2) During this semester, did you use MuPad to help with other assignments or homework?
    a) Never
    b) Occasionally
    c) Frequently
    d) Always

3) Which of the following sections in the Mupad assignment did you find the most challenging?
    a) Complex numbers
    b) Differentiation
    c) Integration
    d) Matrices

4) Overall, do you think using MuPad helped your understanding of the course material?
    a) Not at all
    b) Somewhat
    c) Depends on the topic
    d) Greatly

5) Based on your experience on using MuPad in this course, would you use it in further (non-Mathematics) courses involving calculation?
    a) Never
    b) Sometimes
    c) Depends on the course
    d) Always

6) If you have further comments on MuPad, please write them here.