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Exploring mathematical mindset in question design: Boaler's taxonomy applied to university mathematics

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ABSTRACT

CONTEXT

Dropout from engineering studies at tertiary level remains a persistent global problem. The social psychology theory of mindset explains how behaviour necessary for successful engagement with challenging academic content can be derailed by beliefs about intelligence as fixed-at-birth rather than growth mindset beliefs that intelligence can always be further developed. Given the complexity of research involving humans and the early stage of mindset research in tertiary settings, it is not surprising that the results of a recent systematic literature review on growth mindset interventions in engineering education did not identify a leading intervention. However, the review suggested that growth mindset interventions should address the broader education context and not only individual students.

PURPOSE OR GOAL

Of all subjects, mathematics is one where fixed mindset beliefs are more frequently seen in the general population. High performing students may be at risk from the negative effects of a fixed mindset when they encounter new challenges at university. This research explores the potential of creating growth or fixed mindsets through the words used in mathematics questions. Examples from mathematics assessment tasks will be analysed to see how they align with mindset principles described in a taxonomy by Boaler (2015).

APPROACH OR METHODOLOGY/METHODS

A modified version of the Delphi Technique was used to reach consensus on the applicability of Boaler's taxonomy to undergraduate mathematics courses. Questions from past assessments from first-year mathematics courses were compiled, based on their potential to match the categories in Boaler's taxonomy. In six meetings over three months, all three authors discussed and classified the selected questions into the categories from Boaler's taxonomy. Where questions did not fit, modifications were brainstormed to see if modified questions could align with one or more categories from the taxonomy.

ACTUAL OR ANTICIPATED OUTCOMES

Examples matching all categories of Boaler's taxonomy are presented and contrasted with non-examples on the same mathematics topics.

CONCLUSIONS/RECOMMENDATIONS/SUMMARY

Boaler's taxonomy can guide the design of mathematics questions so that they can also reinforce growth mindset beliefs. Utilising Boaler's taxonomy in addition to the well-established Bloom's taxonomy to guide question setting may increase the possibility of promoting growth mindset. Multiple directions for future research are described.

KEYWORDS

Growth mindsets, intervention, beliefs, assessment, taxonomy, question setting.

Mindset Theory

Dropout from engineering studies at tertiary level remains a persistent global problem (Bengesai & Pocock, 2021). The social psychology theory of mindset (Dweck, 2006; Dweck & Leggett, 1988) explains how behaviour necessary for successful engagement with challenging academic content can be derailed by beliefs about intelligence. The extremes of the spectrum of such beliefs are the 'fixed mindset' belief that intelligence is predominantly fixed at birth and the 'growth mindset' belief that intelligence can always be further developed. Context can affect whether we are closer to one end of the mindset spectrum or the other (Levinthal et al., 2021; Walton & Cohen, 2011). The belief that one is born with a 'math brain' is common (Jonsson et al., 2012) and can be detrimental to students' performance (Rattan et al., 2012). Growth mindsets are typically associated with greater tenacity and success in problem solving (Pierrakos, 2017). Therefore, engineering students may be more successful in their studies if they can be nudged towards the growth mindset end of the mindset spectrum.

In the everyday experiences of engineering students, mindset beliefs are likely to operate more on a subconscious level than a conscious one. Given the complexity of any research involving humans and the early stage of mindset research in tertiary settings, it is not surprising that the results of a recent systematic literature review on growth mindset interventions in engineering education did not identify a leading intervention (Campbell et al., 2021). However, the review suggested that growth mindset interventions should address the broader educational context and not only individual students.

A crucial area that captures students' attention is assessment. The statement by Biggs (1999, 141) remains valid over two decades later: "What and how students learn depends to a major extent on how they think they will be assessed." Those who set assessments may benefit from research on how the words used in assessment tasks may be subtly promoting fixed or growth mindset beliefs. A supportive learning environment should send the message that students can succeed in the academic challenges they encounter.

Mathematics is a subject in which fixed mindset beliefs are more frequently seen (Jonsson et al., 2012) and mathematics educators are likely to encourage ideas about giftedness (Leslie et al., 2015). High performing students may be at risk from the negative effects of a fixed mindset when they encounter new challenges at university. These include avoiding academically challenging work (Mueller & Dweck, 1998), viewing assessment feedback or criticism as a personal attack or an insult (Dweck, 1999), becoming less confident when they put more effort into a task (Miele & Molden, 2010), and being more interested in getting good marks than learning (Dweck, 2000). Furthermore, approximately half of engineering students drop out from engineering studies (Boles & Whelan, 2017) and most dropout occurs in the first year of studies (Lukic et al., 2004). Interventions to develop growth mindsets in engineering students would therefore be well placed in mathematics modules.

This research explores the potential for developing growth or fixed mindsets through the words and approaches used in mathematics questions. It is the first stage in a larger project that will later include feedback from students on reframed questions. The focus of this paper is to establish a framework for designing mathematics assessment questions that align with growth mindset principles.

Boaler's Mathematical Mindset Taxonomy

Boaler (2015) has provided recommendations for writing mathematical problems to encourage growth mindset. These recommendations can be summarised as follows, and we will refer to them as Boaler's taxonomy:

A. Open up the task so that there are multiple methods, pathways, and representations.
B. Include inquiry opportunities.
C. Ask the problem before teaching the method.
D. Add a visual component and ask students how they see the mathematics.

E. Extend the task to make it lower floor and higher ceiling.
F. Ask students to convince and reason; be skeptical.

Boaler's work has focused on school-level mathematics. In this work we explore the practicality of using Boaler's taxonomy in undergraduate mathematics. In line with the guidance that Bloom's taxonomy (Bloom et al., 1956; Krathwohl, 2002) gives to educators when setting assessment tasks, we anticipate that Boaler's taxonomy may help to guide the development of questions that extend the development of students' growth mindsets in addition to developing their mathematical abilities.

Research question

The research question explored in this paper is, to what extent can Boaler's taxonomy be used to guide the writing of university mathematics questions?

Methodology

The Delphi Technique is described by Green (2014, p.6) as "a communication structure aimed at producing a detailed critical examination and discussion." The technique has been used in education research and involves spaced cycles of deliberations by a panel of experts on a problem until reaching consensus or reaching an agreed-upon endpoint. A modified version of the Delphi Technique was used to reach consensus on the applicability of Boaler's taxonomy to undergraduate mathematics courses. The first and second authors compiled 41 questions, 30 from past assessments from first-year mathematics courses they had convened from 2012 to 2020, and 11 from the prescribed textbook for engineering mathematics at our university (Stewart et al., 2016). Questions were chosen for their potential to match the categories in Boaler's taxonomy. In six meetings over three months, all three authors discussed and classified the selected questions into the categories from Boaler's taxonomy. Where questions did not fit, modifications were brainstormed to see if modified questions could align with one or more categories from the taxonomy.

Our backgrounds position us as an expert panel for judging and creating mathematics questions to fit Boaler's taxonomy. The authors have 22, 10 and 8 years of experience teaching and convening first-year mathematics courses. The first author has a PhD on growth mindsets and the second author is working towards a PhD on growth mindsets.

Findings and Discussion

Here we present examples from engineering mathematics assessment questions under each of Boaler's six recommendations.

Open up the task so that there are multiple methods, pathways, and representations

One of Boaler's recommendations is to open up tasks to encourage students to think about different methods and pathways. In the example below, instead of asking, *"Find the fifth roots of* $1 + i$*,"* students are asked to give a visual representation of the solutions.

1. Plot (roughly) all the fifth roots of $1 + i$ on the complex plane below.

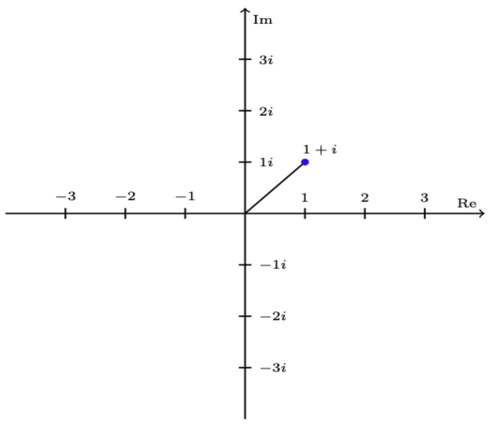

Figure 1: The complex number 1+i plotted on an Argand diagram

This leaves multiple pathways open as the student can perform the calculation through a graphical understanding of roots or using the algebraic methods of finding roots and then plotting them.

Include inquiry opportunities

An example of an inquiry-based approach to assessments is requiring students to do a mathematical investigation. Jaworski (1986) describes mathematical investigations as "contextualised problem-solving tasks through which students can speculate, test ideas and argue with others to defend their solutions." (as cited in Diezmann et al., 2001, p.170). An example of a mathematical investigation problem is outlined below.

2. The Sierpinski triangle is created recursively by removing the middle fourth of each existing triangle as shown below. Let $n = 0$ denote the first (solid) triangle and assume it has sides of length 2 units.

   (a) Show that the first triangle has $Area = \frac{\sqrt{3}}{4}\, side^2$.
   (b) What is the area of the second shape from the left? What is the total length of all the edges of the shape?
   (c) What is the limit of the perimeter and area of the shape as $n \to \infty$?

Figure 2: The Sierpinski triangle

A traditional way of asking this question would be to give the general formula for the area and the perimeter of the n^{th} triangle and ask the student to compute the limit of the area function as $n \to \infty$ as in the example below:

Evaluate the following limits, if they exist.

(a) $\lim_{n\to\infty} \frac{\sqrt{3}}{4}\left(\frac{3}{4}\right)^n$ (b) $\lim_{n\to\infty} 6\left(\frac{3}{2}\right)^n$

Ask the problem before teaching the method

Posing problems for students before introducing the method offers students an opportunity for learning and using intuition (Boaler, 2015). The approach of giving a problem before instruction on how to solve it was shown in a review by Chen and Kalyuga (2020) to be effective for learning the conceptual knowledge of principles underlying procedures, whereas instruction-before-problem was effective for learning procedural knowledge. In this example, a problem about approximating the area under a curve can be asked before the students are taught about Riemann sums and definite integrals.

3. The area under between the graph of $f(x) = 4 - x^2$ and the x-axis between $x = 0$ and $x = 2$ can be estimated using rectangles of equal width as shown in the figure below:

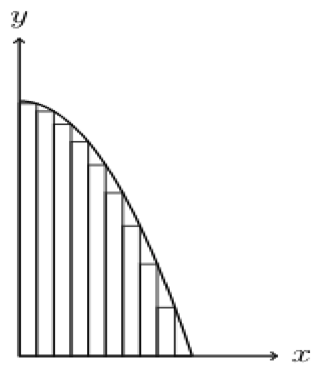

Figure 3: Rectangle of equal width estimating the area under the curve

(a) Let n be the number of rectangles, and let Δx be the width of each rectangle, and x_i be the right end-point of each rectangle. Show that the total area of n rectangles is given by

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x \ = \sum_{i=1}^{n}\left(4 - \frac{4i^2}{n^2}\right)\frac{2}{n}.$$

(b) How can you improve the estimation? How can you find the exact area A between f(x) and the x-axis on [0,2]? Find A using the identity $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

A traditional version of this question could be:

> The Riemann sum for the area under the graph of $f(x) = 4 - x^2$ is $R_n = \sum_{i=1}^{n} f(x_i)\Delta x \ = \sum_{i=1}^{n}\left(4 - \frac{4i^2}{n^2}\right)\frac{2}{n}$, find the area by taking the limit as $n \to \infty$.

This would usually be asked after the students have been taught about Riemann sums.

Add a visual component and ask students how they see the mathematics

The importance of visual representations for teaching and learning of mathematics has been highlighted in several studies (Barmby et al., 2013). Adding a visual component enables students to gain insights into abstract mathematical ideas (Duval, 1999, as cited in Barmby et al., 2013).

In the example below, students are required to understand the relationship between graphs of functions and their derivatives.

4. The figure below shows the graphs of a function f and its first two derivatives, f′ and f″. Which is which?

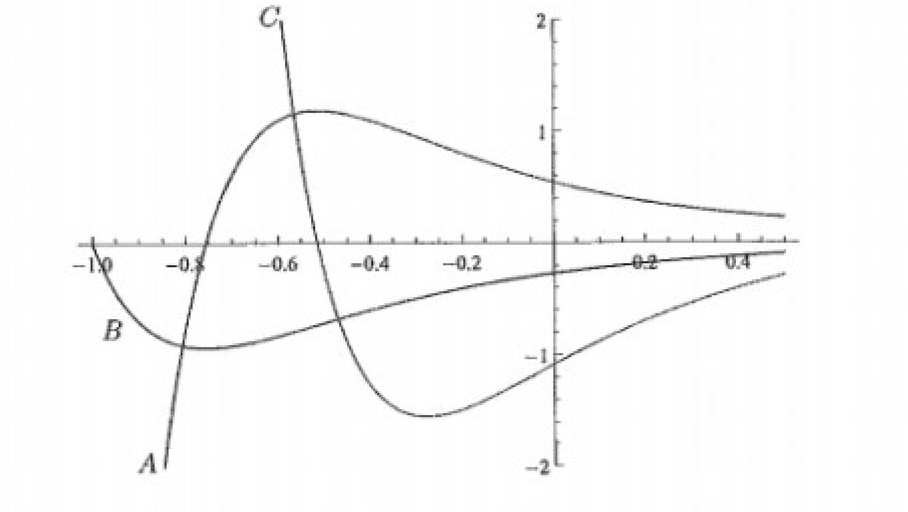

Figure 4: The graphs of f, f', and f''

A traditional version of such a question on the same topic that does not include a visual component would be:

> Given $A(x) = x^3 - 4$, $B(x) = x^5 - 2x^2$, and $C(x) = 5x^4 - 4x$, if one of them is f, another is f' and the other is f'', match A, B and C to f,f',and f''.

Extend the task to make it lower floor and higher ceiling

Low threshold and high ceiling (LTHC) or low floor and high ceiling tasks, as described by Boaler (2015), are tasks that have multiple entry points such that students of all levels can access them. For instance, instead of asking the students to solve the inequality: $|2x - 1| - |x + 3| \geq 8$, the task can be extended as in the example below. This gives the students who may struggle with the inequality an entry point.

> 5. The function f is defined by $f(x) = |2x - 1| - |x + 3|$.
>
> (a) Write f as a piecewise defined function.
> (b) Draw the graph of f.
> (c) Find the set of all x which satisfies the inequality $f(x) \geq 8$.

Ask students to convince and reason; be skeptical

Many researchers have emphasized the importance of promoting reasoning and understanding in tasks (Mueller et al., 2014). Correctly worked examples are an effective method for initial acquisitions of procedural knowledge (Adams et al., 2014). However, Große and Renkl (2007), in their study involving university students, suggested that introducing errors in the learning process can encourage students to reflect on what they know and help them create clear and more complete explanations of the solutions. In the example below, students are presented with an erroneous example, and asked to spot and explain the errors. This gives students an opportunity to offer reasons and critique the argument provided.

6. In the following argument about the function $f(x) = ln(3x^2)$, explain which step is wrong, and what is wrong with it:

Let f(a)=f(b), so $ln(3a^2) = ln(3b^2)$. (1)

Then, using logarithmic laws, we get $ln\,3 + 2ln\,a = ln\,3 + 2ln\,b$, (2)

It follows that $ln\,a = ln\,b$, (3)

So finally, $a = b$ and f is a one-to-one function. (4)

Conclusions and Recommendations for Future Research

In conclusion, each category of Boaler's taxonomy was found to be applicable to university-level mathematics questions. Growth mindsets benefit engineering students by encouraging behaviour needed throughout engineering studies, such as willingness to tackle challenging tasks in which the outcome is not certain and using mistakes and feedback to improve. Mathematics is a core part of engineering, typically taken in the first year of engineering studies when dropout is high. Assessment captures students' attention and designing assessment is a key focus for lecturers. This research has established that mathematics assessments can be designed to align with growth mindset principles.

This finding encourages a number of directions for further research on how growth mindset may be developed through changes to the wording used in mathematics questions. Utilising Boaler's taxonomy in addition to the well-established Bloom's taxonomy to guide question setting may increase the possibility of promoting growth mindset. Future investigations can test the extent to which questions matching the categories in Boaler's taxonomy can help to promote growth mindset in university mathematics students, and if all the categories in Bloom's taxonomy are equally suited to enhancement with Boaler's taxonomy. Future research can also explore how the use of the taxonomy may shift lecturers towards the growth side of the mindset spectrum and help to raise awareness of mindset beliefs that may be conveyed to students in subtle ways.

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