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Applying Benford's Law to Simplify Grading of Engineering Capstone Projects.

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ABSTRACT

CONTEXT

The grading of final year chemical engineering students' capstone subject design reports represents a considerable burden on educators, due to the detailed calculations undertaken by students. The reports represent the accumulation of calculations into material and energy balances, detailed process and mechanical designs, process control as well as economic analysis. To facilitate these calculations, most students utilise computational programs and generate detailed spreadsheets. Hence, the verification of the resulting mathematics behind the generated data requires significant time commitment.

PURPOSE OR GOAL

To simplify this verification for grading, Benford's law was applied here to chemical engineering student capstone project reports. This tool is a statistical method in which data within sets are analysed to detect inconsistencies within the calculations. This enables the rapid identification of features within students' assessments that warrant further attention.

APPROACH OR METHODOLOGY/METHODS

Benford's Law, also known as the significant-digit law, or the phenomenon of significant digits, is the finding that the first numeral of numbers found in series of related records do not display a uniform distribution. Rather, the numeral '1' is the most frequent first-digit in a series of numbers, followed by '2', then '3' and so on. This law also holds for calculations of number sets. As such, the use of this law enables large scale data sets to be quickly analysed to establish if the data has been manipulated or errors exist, and the law is applied in forensic accounting to determine fraud.

ACTUAL OR ANTICIPATED OUTCOMES

Benford's law has been applied to the calculation outputs generated by final year chemical engineering capstone students, to assist in identifying calculation mistakes and errors. The analysis demonstrated that chemical engineering students' capstone project reports followed the significant-digit law. Furthermore, Benford's law analysis determined abnormal student reports that deviated from the law. On further inspect of these reports, the abnormal results highlighted fraud/errors in the students' design calculations, which were not discovered during the standard grading procedure.

CONCLUSIONS/RECOMMENDATIONS/SUMMARY

Applying Benford's law enables a rapid approach to verifying the mathematics within the capstone project reports. This will reduce the time burden on educators, enabling them to focus on ensuring the correct design equations and procedures have been applied.

KEYWORDS Grading; Significant-Digit Law; Design Project; misconduct

Introduction

Chemical engineering capstone design projects are the culmination of a students' engineering degree, where all aspects of their course are brought together into one encompassing design task (Kentish & Shallcross, 2006). To accomplish this project, each student must undertake detailed design calculations around an entire process, as well as major and minor unit operations (Bishop, Nespoli, & Parker, 2012). The wide scope the capstone project means the assessment tasks are ill-defined, to ensure creativity and engineering judgement are demonstrated by each student (Burkholder, Hwang, & Wieman, 2021). The corresponding grading rubrics is therefore ambiguous for educators, to enable flexibility in the design to be emphasized within the overall grade. The central factor in any capstone project are the detailed calculations that utilise engineering principles and equations to solve complex questions. Students provide these calculations for assessment purposes, which can represent hundreds of pages of spreadsheet tables, computer coding and hand calculations. The reviewing and grading of these calculations is an overwhelming time burden on educators, which will become even more onerous with the inclusion of big data, digitisation and artificial intelligence into the chemical engineering curriculum (Chiu, 2021; Pekdemir, Murray, & Deighton, 2006; Scholes, 2021). Strategies are needed to assist educators in grading calculations and importantly to quickly establish those students and the sections of their reports, where errors and misconduct have occurred.

One strategy that has demonstrated potential is the application of Benford's law. This is a statistical tool that can be applied to large data sets to quickly establish if the underlying numbers have been adjusted or altered (Kossler, Lenz, & Wang, 2021). This law is utilised in forensic auditing of financial documents to detect irregularities and fraudulent activity (Azevedo, Goncalves, Gava, & Spinola, 2021; Bolton & Hand, 2002; Cerioli, Barabesi, Cerasa, & Perrotta, 2018; Diekmann, 2007; Nigrini & Mittermaier, 1997). Hence, there is the potential to apply this law to the evaluate chemical engineering student design projects. The objective is to identify areas within the data sets of student reports that are irregular and contain inconsistencies. These areas then become the focus of further investigation to establish how the irregularity developed.

Theory

The significant-digit (Benford) law or the phenomenon of significant digits, is the finding that the first numeral of numbers found in series of related records do not display a uniform distribution (Newcomb, 1881). Rather, the numeral '1' is the most frequent first-digit in a series of numbers, followed by '2', then '3' and so on. For example, the numeral '1' will be the first-digit 30.1% of the time. This law also holds for calculations of number sets. The first significant-digit law is satisfied for a set of numbers of base 'b', if the leading digit (d) occurs with a probability of (Hill, 1995; Varian, 1972):

$$\mathrm{P(d)} = \log_{\mathrm{b}}\left(1 + \frac{1}{\mathrm{d}}\right) \tag{1}$$

For a base of 10 (e.g., 1, 2, …., 9) the first-digit law probability is therefore:

$$\mathrm{P(d)} = \log_{10}\left(1 + \frac{1}{\mathrm{d}}\right) \tag{2}$$

The proof of this law is derived from number sets being the combination of distributions. Hence, the law holds when higher order mathematics (multiplication, division, logarithms, etc) are applied to a data set.

The law also applies to the second numeral of a number, and any given digit in a number, with the probability of encountering a number starting with a string of digits (n) being (Hill, 1995):

$$\mathrm{P(n)} = \log_{10}\left(1 + \frac{1}{\mathrm{n}}\right) \tag{3}$$

This enables the second and third-digit frequency to be established. The probability of the first- and second-digit frequencies are provided in Table 1.

Table 1: Probability of base 10 numerals being the first and second- digit of a number, as defined by Benford's law.

Digit	1st	2nd
0	-	0.120
1	0.301	0.114
2	0.176	0.109
3	0.125	0.104
4	0.097	0.100
5	0.079	0.097
6	0.067	0.093
7	0.058	0.090
8	0.051	0.088
9	0.046	0.085

Important for this research, is the knowledge that deviations from Benford's law for large scale data sets are an indication that the underlying data may have been manipulated or errors exist (Todter, 2019).

Experimental

Chemical Engineering student design project reports were analysed to determine if individual student report's calculations deviations from the expected Benford's law outcome. The reports were all processed by optical character recognition software (Adobe Acrobat Pro) to identify numbers in both printed and handwritten format. This process was based on electronic submissions, through the university's learning management system. The digitised data sets were subsequently analysed for all hand, spreadsheet outputs and computer coding calculations. Numbers in drawings, standards and references were discounted, as these do not represent calculations. The removal of this information was necessary to enable the Benford's law analysis to be focused on calculation endeavours. Hence, this reduces the detection of false positives.

The analysis took under 10 minutes per report, with most of the time associated with excluding numbers that were contained within drawings and quoted data, which do not represent calculations.

This analysis was undertaken on five years of student capstone design project reports, that covered a wide range of chemical engineering topics, including camel milk processing, rare earth element refining, vitamin production from natural sources, helium extraction from natural gas and silicon ingot refining.

Results and Discussion

To establish that Benford's law can be applied to chemical engineering student design projects, the cumulative digit frequency in student reports based over the years analysed is provided in Figure 1 for the 1st digit and Figure 2 for the 2nd digit. Included in these figures is the expected probability arising from Benford's law. The analysis indicates that the frequency distribution approaches that of Benford's digit law probability the greater the number of reports analysed. Hence, this verifies that Benford's law is obeyed in chemical engineering student design reports for the first two digits, and therefore deviations from the expected probability in student reports warrants investigation.

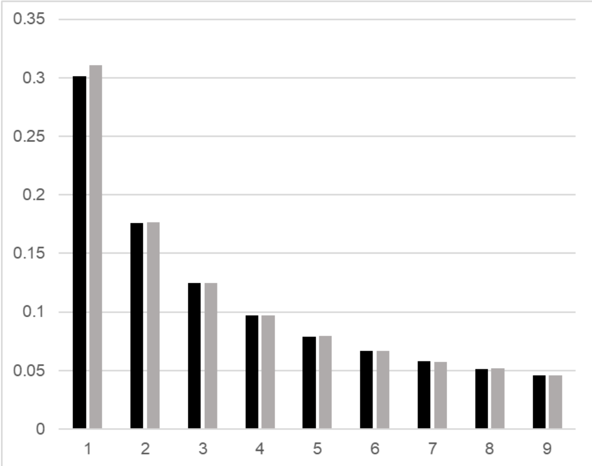

Figure 1: First-digit frequency distribution of student design reports analysed (grey) compared to the Benford's law probability (black).

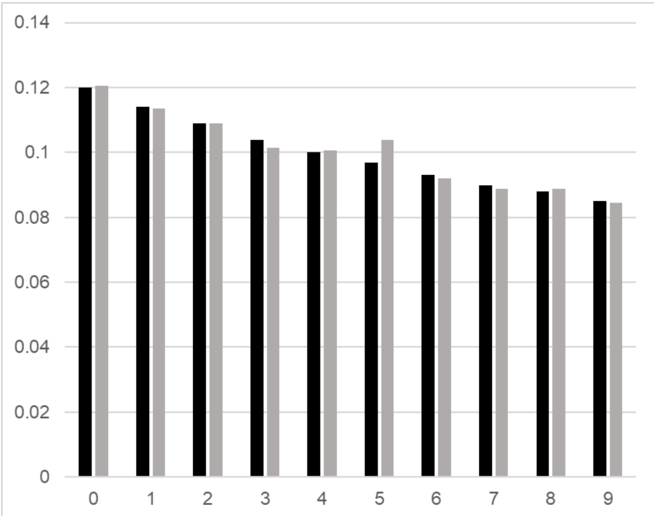

Figure 2: Second-digit frequency distribution of student design reports analysed (grey) compared to the Benford's law probability (black).

The cohort of students' distribution approximates the distribution expected for the 1st digit, with the numeral '1' frequency dominating the distribution. This analysis represents 405 reports and over 5 million numbers. Hence, this demonstrates an underlying fact of the Benford's law, large data set are required to achieve the probability frequency that matches the law. This is the reason why only large assessment items, such as capstone projects, are suitable for this type of analysis. Smaller assessments that are solving only one problem will rarely have enough calculations to represent a viable base for analysis.

The frequency distribution for the 2nd digit is less clear and required all 405 reports to be analysed for the student design projects distribution to match the 2nd significant-digit probability. There is some variability in the numeral '3', '4' and especially '5'. This corresponds with the practical application of Benford's law, where there is strong alignment for data sets with the first-digit, but the frequency of the 2nd and 3rd digits are less well aligned.

Benford's law analysis identified several student reports that had frequency distribution profiles that deviated significantly from expectations. Examples of these are provided in Figures 3, 4 and 5. Detailed inspection of these student reports indicated serious issues and concerns regarding the design outcomes and calculation justifications presented by the students. For two of the reports submitted, the underlying coding the students used had faults that resulted in calculation errors. In both cases this was due to the transferring of Microsoft Excel scientific numbering into student written software. Excel used the 'E' notation to designate a scientific number, and directly transferring the text version of this number retains this configuration (e.g. 1.2345E-01). The subsequent programmed software used by the students did not understand this number format and subsequent discounted the entered number in the resulting calculation. For one student report this resulted in the first-digit distribution favouring even digits (Figure 4) and for another student report this favoured the digit '3' (Figure 5). To identify and determine these errors based on just direct reading of the reports and ancillary information would require a very focused educator to pick-up this error. Importantly, Benford's law analysis enabled these reports to be flagged quickly, which then allowed a more detail inspection by educators to determine the problem. It is a concern that some chemical engineering students are not aware of the issue of number formatting, as well as the lack of checking of their work. The rapid evaluation of this issue represents a significant work and time saving to educators.

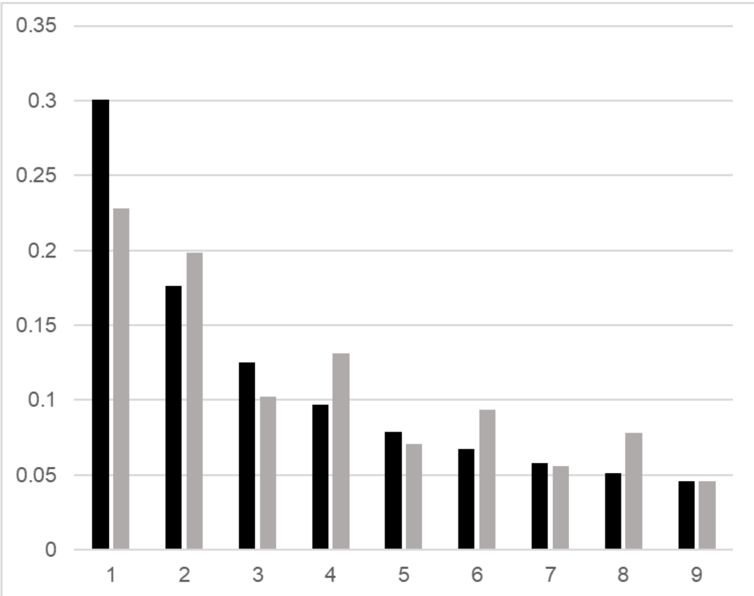

Figure 3: First-digit frequency distribution of a student design report analysed (grey) compared to the Benford's law probability (black).

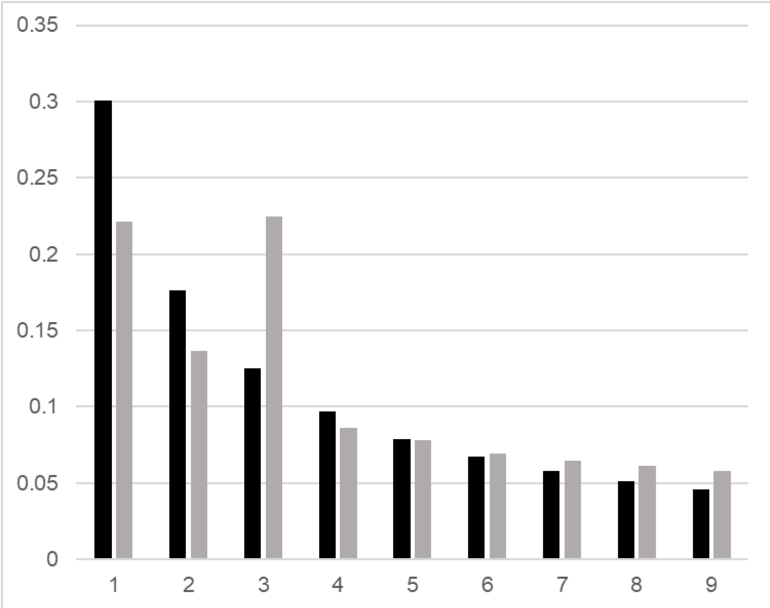

Figure 4: First-digit frequency distribution of a student design report analysed (grey) compared to the Benford's law probability (black).

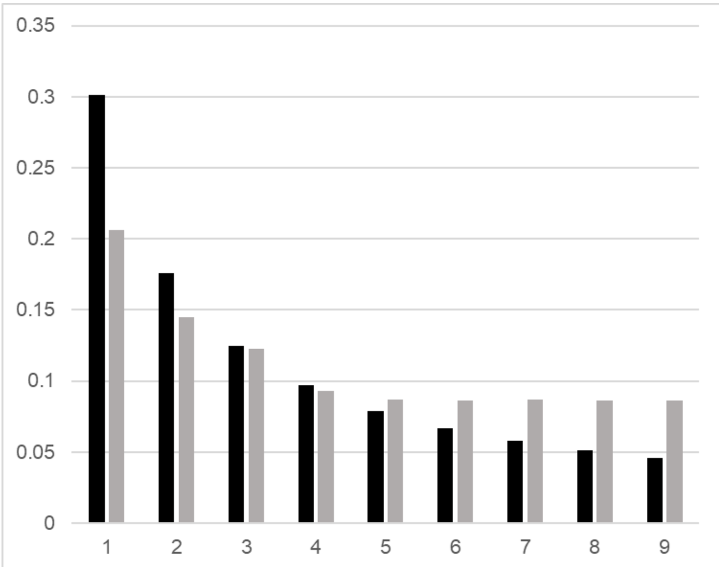

Figure 5: First-digit frequency distribution of a student design report analysed (grey) compared to the Benford's law probability (black).

For one student, the significant-digit distribution had a profile that was more evenly distributed across the digits than Benford's law probability (Figure 5), which upon investigation by educators revealed that design parameters between the material and energy balances, reaction kinetics and detailed process design were completely different. Specifically, the student undertook one design for their material and energy balances, these parameters were subsequently not used to solve their reaction kinetics problem, and both of those solutions were not used to design the process unit. Essentially the student presented three different designs, where the calculation bases for the three sections of the report were completely different. As such, the level of calculations undertaken in the three sections were limited, meaning the data set presented was not extensive and the resulting frequency distribution demonstrated this fact. Hence, the significant-digit analysis and comparison with Benford’s law highlighted there was a problem with the student’s report and subsequent investigation revealed the error.

Conclusion

Benford’s law is a statistical tool that can be used to assess engineering students’ assignments for error when these assignments contain significant volumes of calculations. This tool has been applied here to chemical engineering capstone design projects, which have been verified to obey the law’s probability of first significant digit distribution. Importantly, the analysis identified several student reports that contained errors, due to the individual student’s distribution deviating significantly from the expected frequency distribution. An educator would need to undertake in-depth analysis of these reports to come to the same conclusion, which Benford’s law analysis can highlight in a much shorter time. Hence, this approach represents a time saving tool for educators to evaluate student assessments.

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