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Enhancing Engineering Pedagogy with Near Metaphors

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ABSTRACT

CONTEXT

Engineering education faces competing challenges of resource limitations due to scale, time pressures for research, costs, and changing student backgrounds with quality demands, as promoted by using active learning pedagogies to enhance learning outcomes. In mathematically oriented engineering courses in electronics and telecommunications engineering, such as signal processing and probability, the dilemmas faced come to the fore. This is demonstrated by the challenges students face in problem solving and acquiring threshold concepts. This is epitomised by the recent finding that short videos for mathematics instruction may target easy memorability but fail to address deep mathematical concept development (Almaric *et al*., 2022).

PURPOSE OR GOAL

This paper aims to develop pedagogical methods that more effectively target the development of long-term capacities for learning and skills acquisition in the context of mathematically oriented engineering courses. Building on the close link between embodied cognition and the numerical and spatial cognitive aspects of mathematical thinking, this paper examines the effectiveness of metaphors and analogical thinking to aid in problem solving and acquiring threshold concepts.

APPROACH OR METHODOLOGY/METHODS

The focus is on the application of near analogical connections as they are more often used in generating hypotheses and explaining unexpected findings than far analogical connections, which are more often used to communicate findings. However, near analogies are more difficult to discern, and a systematic approach for identifying them was achieved using linguistic theory. Class exercises and assignment questions were created using metaphors for the comprehension of advanced content and problem solving.

ACTUAL OR ANTICIPATED OUTCOMES

Student responses were evaluated for depth of understanding, semantic usage, problem representation, and concept integration. A coding scheme was used to identify different levels of performance. A subsequent assignment subjectively tested the ability to handle more extended arguments. The results suggest that the students developed more flexible ways of thinking about complex mathematics and gained insight into difficult-to-understand threshold concepts.

CONCLUSIONS/RECOMMENDATIONS/SUMMARY

Metaphors and analogical thinking provide an effective method for students to progress to more advanced levels of study and improve the quality of understanding. More generally, analogical thinking applies to many engineering areas beyond mathematically oriented courses, and awareness of their usefulness will have long-term benefits.

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KEYWORDS

Threshold concepts, metaphors, analogical thinking, cognition, deep learning

INTRODUCTION

Engineering educators face competing challenges of resource limitations due to scale, time pressures for research, costs, and changing student backgrounds with quality demands, as promoted by using active learning pedagogies to enhance learning outcomes. In mathematically oriented engineering courses in electronics and telecommunications engineering, the dilemmas faced come to the fore. This is demonstrated by the challenges students face in problem solving and acquiring threshold concepts. The aim of this paper is to develop pedagogical methods that more effectively target the development of such important long-term capacities for learning and skills acquisition in the context of mathematically oriented engineering courses where deep understanding is required to solve complex problems creatively, advance the theory, and become effective designers.

Building on the close link between embodied cognition and the numerical and spatial cognitive aspects of mathematical thinking, this paper examines the effectiveness of metaphors and analogical thinking to aid in problem solving and acquiring threshold concepts in mathematically oriented engineering courses. Metaphors and analogies express meaning in one domain, usually the more complex and less understood domain, in terms of meaning in another domain that is simpler and better understood. Analogical reasoning and metaphors foster creativity since they help to develop connections between unconnected or distantly connected concepts and prove useful for transferring knowledge of one scientific object to a new one (Abraham, 2018). Near analogical connections occur in the same domain, whereas far analogical connections occur cross domains. Experts in the field use near analogies more often than far analogies for creative thinking, generating hypotheses, and explaining unexpected findings. In contrast, far analogical connections (e.g., between economics and biology) are often used to communicate findings to colleagues or the general public. The generation of hypothesis and understanding of discrepancies is fundamental to understanding and developing models, which is an important skill for engineers.

The focus in this paper is on applying near analogical connections and the primary research question is: How can near metaphors be used to improve deep learning and problem solving in mathematically oriented engineering courses? The perspective of cognitive processes and cognitive modelling is used to frame the analysis and discussion in the paper. A challenge in using metaphors is that they can be difficult to discern. The paper addresses this issue by using linguistic theory to develop a systematic approach to identifying near analogies. The metaphor approach was implemented in two courses where class exercises and assignment questions were developed to aid understanding of advanced content and problem solving. The results suggest that the students developed more flexible ways of thinking about complex mathematically oriented content and gained insight into difficult-to-understand threshold concepts. The rest of the paper begins with the context and background on active learning, threshold concepts, cognitive models used, and metaphors, followed by a description of the linguistic approach to metaphors, the application of the metaphor approach to engineering courses with results, and a discussion.

CONTEXT AND BACKGROUND

When faced with difficult problems in mathematically oriented courses, students often feel powerless or see problem solutions that appear to be based on tricks or come from ‘thin air’. Two ways in which problems can become difficult are as follows. First, the problem may require a more creative approach to the solution, requiring more divergent and flexible thinking. Second, the problem requires a broader range of background knowledge in mathematics. The metaphor concept, by exposing links to more distant concepts, helps in solving both problems. Threshold concepts can assist by increasing understanding and being powerful tools for problem solving. Threshold concepts are those core concepts which are transformative since once they are obtained, they open up new ways of understanding (Meyer & Land, 2005). The properties of threshold concepts include that they are troublesome, transformative, irreversible (difficult to unlearn), and bounded (having distinct and particular purposes). Concepts are 'troublesome' if they are difficult to understand or accept because they are tacit, inert, use unfamiliar language, or evoke fear of uncertainty (Male & Bennett, 2015).

The state-of-the-art pedagogical approaches for fostering problem-solving skills and developing understanding of threshold concepts are currently strongly based around various active learning approaches. This is generally defined as one where the learner is actively involved in constructing knowledge by doing "meaningful learning activities and think[ing] about what they are doing" (Prince, 2004). The highly cited (7900+ citations in 2022) meta-analysis study by (Freeman et al., 2014) of 225 studies of science, engineering, and mathematics learning showed that active learning, compared to traditional lecturing, improved pass rates and concept understanding and was generally more effective. However, the successful application of active learning faces significant challenges that limit understanding and problem-solving ability. Poor active learning design can lead to meaningless activities and doing without thinking. As (Hartikainen, Rintala, Pylväs, & Nokelainen, 2019) observe, "adopting a specific activity does not ensure that the activity is constructivist", for example, when step-by-step procedures are given to students who follow them without critical awareness (Browne, 2017). hner, Sweller, and Clark (2006) found that students trained using an active approach tended to give more elaborate, less coherent, and erroneous explanations and bring in more irrelevant information during problem solving. Once knowledge is contextualised, separating it from practice knowledge may be difficult and learners cannot generalise, make abstractions and transfer knowledge to new situations. There may be more reliance on backward reasoning and less use of forward reasoning, a feature of expertise. This paper seeks to understand such paradoxical outcomes and to use the concept of near metaphors as means of addressing some of the pedagogical problems involved. An understanding of cognitive processes is important in developing suitable pedagogies. As motivation consider how, with increasing technology use, videos have become widely used in courses. Principles for designing effective videos are given by (Dart, Cunningham, & Gregg, 2022). Yet, a recent finding is that short videos for mathematics may fail to address the development of deep mathematical concepts because teaching material targets easy memorability which engages domain-general short-term episodic memory rather than the domain specific areas of the mind used for deeper cognitive processing (Amalric, Roveyaz, & Dehaene, 2022). Also, despite a significant amount of literature on characterising and identifying threshold concepts, it is harder to locate sources that explain, in cognitive terms, why threshold concepts are difficult. Using the metaphor approach as a basis, an aim of the paper is to obtain a deeper understanding, at a cognitive level, of problem solving and learning threshold concepts in the engineering education context.

Analogical reasoning using metaphors

The role of metaphors and analogies in scientific thinking and problem solving is widely recognised. For example, the collection of papers (English, 2013) is entirely devoted to the use of metaphors and analogies in mathematics. Most of the analogies used by scientists to generate hypotheses and explain unexpected findings were from a similar domain. When understanding experimental procedures with a single unexpected finding, analogies were often drawn between one experiment and a similar experiment. In this case, the analogies were based on superficial features such as objects' similarity and properties. When formulating a hypothesis (multiple unexpected findings), the analogies relied less on superficial features and more on structural and relational features. Metaphors may help reveal this deeper structure. An example in biology is how knowledge of how steroid hormones aid biological reproduction led, by analogy, to the discovery that steroids could stimulate mould growth (Knorr-Cetina, 2013). Scientists, as analogical reasoners, can achieve conceptual interaction and the extension of knowledge using analogical thinking. Interaction with one side of an analogy assists in building models on the other side of the analogy, e.g., considering proteins as 'sand' with hardness and size may suggest the following reasoning (Chapter 3 (Knorr-Cetina, 2013)):

"If the protein looks like sand … it must be denatured".
"If it *is* denatured, its effect would be to dilute the samples, and nothing else".
"If it does dilute the samples just like sand, it would prove the 'dilution theory' in which everybody believes."
"But if it does *not* have same effect as sand, I can finally disprove this dilution rubbish and propose my own interpretation."

Cognitive aspects of metaphors

Metaphors and analogies facilitate problem solving and design in multiple ways. We use the insight cognitive problem-solving model of (Eysenck & Keane, 2005) as a framework for analysis and to provide insight in threshold concepts. The components of this model are shown in Figure 1. Problem representation determines the features of a problem that are required to obtain a solution. This representation is produced using perceptual information and prior knowledge and can be thought of as having an essential symbol structure (e.g., tree of nodes), which provides redundancy to predict parts of the structure that have not been searched. The problem representation is searched using heuristics to find the solution. If a solution is not found, then an impasse occurs. This is followed by a change in problem representation, and the process is repeated until a solution is found. Representational change also uses prior knowledge and perceptual processes. Typical forms of representational change include constraint relaxation, reencoding, where some aspects of the problem representation are reinterpreted, and elaboration in which new problem information is added (Eysenck & Keane, 2005). Design has many features in common with problem solving. However, the mental representations are usually more ill-structured, and the goal state is open-ended (Alexiou, Zamenopoulos, Johnson, & Gilbert, 2009). This requires making connections between more distantly related concepts using divergent and flexible thinking.

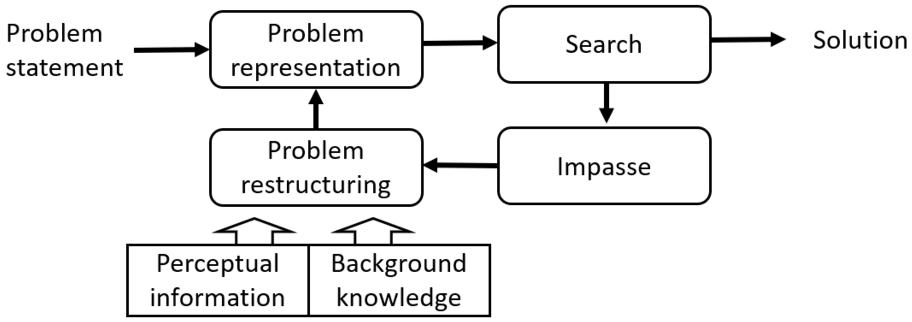

Figure 1: Insight problem-solving model (Eysenck & Keane, 2005)(reformatted)

Cognitive functions used in metaphoric and analogical thinking include integrating visio-spatial representation and relations, manipulating relational information, inhibitory control, semantic memory, semantic retrieval and inhibition, integrating relations, and remote search. Analogies involving associations (e.g., A and B appear together) are often easier to use than those based on categories (e.g., A and B are both C), since association involves direct perceptual aspects, while categorisation uses semantic knowledge. Abstraction aids problem solving by facilitating the search for solutions in the problem representation.

Metaphors link concepts to each other and support concept development because they can be used to support the production of novel combinations of properties and fill gaps in terminology when there are no existing terms. Conceptual change can occur through the restructuring of concepts, theory replacement, and conceptual elaboration (Tolmie & Selma, 2020). Restructuring concepts involves splitting, merging, or altering the core aspects of concepts to increase coherence. Theory replacement involves the substitution of old theories with new ones when the inadequacy of old theories becomes apparent. Conceptual elaboration occurs when existing concepts are used in new areas in concepts being linked together and refined, and the development of new languages and symbols (Hu, Shealy, Grohs, & Panneton, 2019).

Metaphors can support abstraction by linking linguistic and nonlinguistic aspects of phenomena (Wilson-Mendenhall, Simmons, Martin, & Barsalou, 2013). For example, the abstract social concept of “convince” includes intentions, beliefs, internal states, affect, and actions of the self and others. Abstraction aids problem solving by facilitating the search for solutions in the problem representation. For example, in the area of clinical medicine, statements made by patients are converted into more abstract problem representations that trigger clinical memory and allow related knowledge to be located for use in clinical reasoning (Bowen, 2006).

Furthermore, there is dissociation of linguistic and nonlinguistic processes in the mind. Studies have shown a dissociation between numerical processing and language processing (Amalric & Dehaene, 2016) (Nieder, 2019). For example, number sense is processed separately from rote facts (e.g., times tables) that are encoded in linguistic form. It has also been found that language syntax and arithmetic syntax functions are distinct since subjects with impaired language functions can still judge the equivalence of algebraic notation and simplify algebraic expressions. Mathematical problem solving involves the semantic system through the use of mathematical conceptual knowledge, terminologies, facts, rules, and principles. Arithmetic computation and mathematical problem solving are also dissociated with each making independent contributions to mathematical achievement (Zhou et al., 2018). The recruitment of mathematical problem solving and language processing varies, depending on the type of problem (Liu et al., 2017), with language processes used more in the early stages of problem presentation and less in reflection (Amalric & Dehaene, 2016). The dissociation between mathematics and language can be experienced by the difficulty of putting mathematical ideas into words. We postulate that some of the difficulty of threshold concepts is due to the abstraction, nonlinguistic aspects, and dissociation factors described above.

LINGUISTIC MODEL OF METAPHORS

Near analogies are sometimes difficult to detect. The following summarises a linguistic approach to metaphor study to facilitate a more rigorous approach to defining and identifying metaphors (Goatly, 1997). In linguistics, metaphors are distinguished from literal text where the meaning is determined by the usual meaning of words. Metaphors are characterised by nonconventional usage, a level of contradictoriness, and greater processing and interpretive effort. Analogical reasoning can be distinguished from metaphors. Analogies have a stricter relational structure of the form A is to B as C is to D, whereas with metaphors, the association is more varied and flexible. From a grammatical perspective, the components of a metaphor involve the following:

- The topic (sometimes called the target) is the primary object of interest.
- The vehicle (sometimes called the source) is something from another domain that provides information about the topic.
- The ground that specifies the basis on which the vehicle is related to the topic.

Not all components of a metaphor need to be present. However, the vehicle must be present, and either the topic or the grounds may be left unstated. The surrounding text may also contain a signal indicating that a metaphor is present and that the text cannot be interpreted literally. The signal may be lexical or grammatical. Metaphors serve a range of functions. The following functions described by (Goatly, 1997)(Table 6.3) are the ones most applicable to this paper.

Gap-filling	• A lexical gap when no suitable term exists for a new object: • Approximation: Existing terms are interpolated to better express meaning. • Precision: Used to increase the accuracy of a quantity or process by modification of terms
Explanation	Used for an explanation with reference to more familiar entities.
Modelling	Metaphors can be used as scientific models and theories. Helping to make predictions that more rigorous methods can test.
Reconceptualisation	Assists in theory building by viewing experience from a different perspective and undoing existing categories.
Argument by analogy	By reasoning in the vehicle world, the topic world is better understood.

Goatly (1997) describes syntactic resources in language that can be used to signal metaphors, identify topics, and identify grounds. For example, metaphors can be signalled explicitly (e.g., the text contains the word "metaphorically"), or implicitly using intensifiers (really), hedges (a bit), semantic metalanguage (both), mimetic terms (model, plan, image), superordinate terms (kind of), copular and similes (like, as), comparisons, perceptual processes (seemed), etc. Topics can be specified using a copula (e.g., the eye is a teardrop), apposition (the eye, a teardrop), genitive (the teardrop of an eye), noun premodifier (the teardrop eye), etc. The reader is referred to (Goatly, 1997) for more details. Examples of near metaphors pertaining to the courses below include:

A Fourier series is a model for a Fourier transform. As the period increases, a continuous limit is approached.	The vehicle is "Fourier series", the topic is "Fourier transform", the signal is "a model for" and the grounds is "As the period increases, a continuous limit is approached". Use of the metaphor allows results in the Fourier series domain to suggest comparable results in the Fourier transform domain.
The exponential distribution is the continuous version of a geometric distribution.	The grounds (both distributions have the memoryless property) are absent. The metaphor may suggest that the rate function of both distributions is the same.
Parseval's equality equates power in the time domain with power in the frequency domain.	In this case, an equal sign (=) in Parseval's equality formula signals the metaphor. If computation of power is difficult in one domain, then the metaphor (as an identity) may provide, through a problem restructuring, an easier means of computing power in the other domain.

METAPHORS USED IN TWO COURSES

Random processes and probability course

Exercises based on metaphors and analogies were developed for use in a third-year undergraduate course on random processes in a telecommunications engineering programme to improve problem solving. This course covered fundamental probability models, random variables, random vectors, random processes, and spectral density. Important threshold concepts in the course include multivariate probability and spectral density functions. The metaphor exercises were part of a pilot study to test the suitability and effectiveness of using the metaphor approach. The approach was introduced to the students using a short slide presentation that was immediately reinforced with interactive activities that involved the entire class. To simplify the explanation to the class, the use of metaphors was described as one that allows the same thing to be conceived in two different ways. Students then carried out tasks using metaphors independently as part of a formal multi-question assignment. The questions were designed to be a low-risk task for the students. Responses were not assessed for the correctness of the answers, but for the degree of completion. The general format of the metaphor-related questions was to have a short extract from theory or a more advanced application with associated tasks. Such extracts provide a more authentic source of material that is relevant to the specialisation. The questions targeted near metaphors and were centred on threshold concepts. Two separate assignments each had a question on metaphors: Assignment 2, in the middle of the course and the second in Assignment 4A, towards the end of the course.

- In Assignment 2, the extract was on the derivation of the covariance of a transformation of a Gaussian random vector. This targeted the multivariate probability threshold concept.
- The question in Assignment 4A involved understanding the Hilbert transform in communications and targeted links to existing knowledge.

A second set of assignment questions was designed to assess how well the students understood the problem solving and more complex lines of argument.

- Assignment 3 used an extract on tracking and estimation using least squares techniques.
- Assignment 4B used an extract on mean-square filters using spectral density analysis in which students were required to fill in the missing steps in the derivations.

Both of these questions challenged the limitations of linear thinking, where mathematical derivations are conceived as sequential transformations (A is transformed to B, etc.) and concepts are dealt with one at a time. In contrast, metaphors facilitate the parallel consideration of multiple concepts (both A and B) where there is a dialogue between concepts.

Data analysis

The data used for the analysis were the student's written responses to the assignment questions of the 13 students in the course (approximately 32 pages of material). For quantitative analysis, the responses were coded using criteria based on the theory of metaphors and cognitive processes given above. For the metaphor questions in Assignment 2 and Assignment 4A the coding criteria were:

- Criteria (1): Explanation of the metaphor grounds. This was classified as full, partial, or no explanation of the relationship between the vehicle and the topic.
- Criteria (2): Level of semantic usage. This was classified as full, partial or none, depending on how much mathematical semantic knowledge was used.

The responses to the problem-solving questions (Assignment 3 and Assignment 4B) were coded using the criteria:

- Criteria (3): Problem representation. This was classified as full, partial, or none.
- Criteria (4): Degree of use of parallel concepts. This was classified as parallel or sequential.

Regarding criteria (3), a student response only gives the end solution, that is, the result of the search. Usually, the response does not explicitly describe the mental representation of the problem used by the student and this must be inferred. For similar reasons, the search heuristics usually must also be inferred. Making judgments regarding criteria (1)-(4) requires a deep understanding of the subject. The author is well qualified to conduct this ranking, having over twenty years of experience in teaching and research in mathematically oriented telecommunications content, including signal processing, probability, and communications theory.

The results of coding the written responses are shown in Table 1 for the metaphor questions and Table 2 for problem-solving questions. Across all questions analysed, there was a range of answers. However, correlations between one criterion and another were observed. These correlations were determined for this small sample size by comparing the ranks of each student. In Table 1, metaphor grounds were positively correlated with the level of semantic usage in the metaphor question suggesting a relationship between broader knowledge and a greater capacity for abstracting commonalities between concepts which can assist with finding metaphors. In Table 2, students with higher scores at the level of parallelism were positively correlated with better problem representation, suggesting that the incorporation of a variety of aspects into a concept through parallelism can help find ways around problem obstacles. A limitation of this analysis is that the data was taken from an operational course where other teaching techniques could be confounding factors. For a given criterion, factors that may have led to a lower ranking were the ability of the student, the novelty of using the metaphor concept and the more challenging nature of advanced material. Determining the causality between the criteria was also difficult for similar reasons. Some students did not attempt to answer the questions. This affected assignments 4A and 4B more than the others. One explanation for the non-answer was that these assignments were in the latter part of the course when students were under greater time pressure and the tasks, seen as extensions, had lower priority.

	Assignment 2		Assignment 4A	
	Grounds	Semantic usage	Grounds	Semantic usage
Full	4	3	3	1
Limited	5	6	1	2
None	1	1	0	1
No answer	3	3	9	9

Table 1: Metaphor analysis results

	Assignment 3		Assignment 4B	
	Problem representation	Level of parallelism	Problem representation	Level of parallelism
Full	2	2	3	3
Limited	5	6	4	3
None	1	0	0	1
No answer	5	5	6	6

Table 2: Problem-solving results

Assignment 2	Assignment 4A
Assignment 2: (Find three instances where the metaphor concept … is used. In each case, explain the connection between the two objects/processes involved): *“connection for Gaussian random variable”* *“if we want to see some specific result, we would therefore transform on into the other”.* *“X is also a Gaussian vector then Y=AX+b is also a Gaussian random vector.”* *“The marginal pdfs [probability density functions] have similar patterns.”* *“Is another metaphor mathematically as an identity of something is similar to being the same as the theorem it identified.”*	Assignment 4A (Using the idea of metaphors, describe how the complex baseband signal can be used to explain features of the transmitted signal?). *“used to present the information of the transmitted signal … Because it has a real part and imaginary part. From these, we can get the phases, frequency and amplitude.”* *“A baseband signal is a signal emitted that directly expresses the message to be transmitted.”* *“This property is very valuable in simulation … we no longer have to do simulations at carrier frequencies.”*

Figure 2: Extracts from student answers on metaphors

Assignment 3	Assignment 4B
Assignment 3: (Identify important parts of theory that are used in the extract) *“properties of joint distributions”* *“independence … can be helpful”.* *“Basic expectation and variance identities.”* *“Estimate X from observed value of Y – Bayes’ Theorem.”* *“Linear Least Squares Estimates is also using the equations ... used in most of our lessons.”*	Assignment 4B (For the following three sequences of derivations... identify in earlier work in lectures and elsewhere where this type of sequence has been used): *“Related to week 8 correlation & cross-correlation (with associated mathematical derivation)”* *“based on … which I learned from middle school and lecture of expectations”.* *“The values of all the elements in the impulse response sequence cannot be obtained so the main solution methods needs to obtain the frequency response of the system from the frequency domain first.”*

Figure 3: Extracts from student answers on problem solving

Qualitative insight was obtained by examining student written responses. The extracts in Fig. 2 from the metaphor exercises show an awareness of thinking about mathematical entities in more than one way and the usefulness of doing that, although reasons were not always provided. Some students viewed metaphors as sequences of transformations, while others focused on parallel aspects such as patterns, which may contain nonlinguistic information (e.g. between probability distributions) and identities, which reveal structural equivalences. Some students described the transfer of properties, such as Gaussianity, across mathematical transformations, which supports analogical reasoning. Such ways of thinking can address difficulties in understanding threshold concepts, e.g., as in Assignment 2 when expanding familiar univariate distributions to more conceptually rich multivariate distributions. The use of complex numbers to represent phase, frequency, and amplitude described in Fig. 2 integrates multiple representations that can used for analogical reasoning. Extracts from the problem-solving questions (Fig. 3) show awareness of a range of topics covered earlier in the course, knowledge external to the course, and the ability to identify specific technical knowledge in abstract form (e.g., independence, correlation function, convolution, impulse response), which help with searching the problem space. The final response describes how moving between the time and frequency representations of a signal can be used for problem solving.

Signal processing course

The metaphor approach was also used to develop material for a course on Fourier analysis of continuous-time signals. Two significant threshold concepts for signal processing are time-frequency transformation and discretization (Male, Togneri, & Jin, 2021). The current paper describes how the metaphor approach was used to address difficulties in understanding the first threshold concept: the representation of signals in the frequency domain. The metaphor approach was to devise a method for conceptualising angle, which is central to the concept of frequency, in two separate ways. In particular, the angle can be considered as the distance along the circumference of a circle. This links temporal and spatial representations and abstracts the concept of movement, allowing concepts in the time domain to be transferred in concepts to the frequency domain. For example, it can be used to explain signal aliasing, which refers to shifts in frequency caused by sampling, by referring to how the same point on the circumference is reached using a single traversal or multiple traversals around the circle. Potentially, other concepts that can be integrated using different cognitive aspects related to frequency include magnitude, comparison, fractions vs. decimals, the arrival time of moving objects, spatial information, periodicity, energy flow (wave radiation), and tools such as oscilloscopes (Mason & Just, 2016). This course was taught online with an overseas institution. In-depth analysis of student understanding was difficult, although feedback from the overseas course convenor was positive.

DISCUSSION

For both the probability course and the signal processing course, near metaphors provided a useful tool for teaching threshold concepts by exposing the deeper structure of concepts and facilitating concept development by relating novel concepts to more familiar ones. In the random processes course, the use of metaphors helped in the teaching of the threshold concepts of multivariate probability (extending the concept of univariate probability) and spectral density functions (through concept elaboration, building on Fourier analysis), which assists in understanding the subject of random processes, which, in turn, supports the deeper analysis of signal transmission (e.g., signal errors). Near metaphors can assist by filling in gaps of terminology, suggesting models, and providing explanations. The use of metaphors enriched concepts by integrating various aspects. For example, spectral density functions integrate concepts of frequency, probability functions (mathematical knowledge), periodicity (time, motion), Parseval's theorem (energy) etc. Often this includes semantic, and nonlinguistic aspects when mental concepts cannot always be easily verbalised. By looking for grounds which explain transfer between and topic and vehicle, metaphors can with help abstraction and determining patterns. The use of metaphors challenged students linear thinking when faced with long derivations and helped them to move towards parallel thinking, allowing more flexibility of thought. It provided a valuable method for developing creative problem-solving skills by facilitating problem representation and search. The use of metaphors for problem solving is transformative because it opens up new ways of solving problems, is irreversible because it is hard to unlearn once experienced, and is integrative because it enables better use of prior knowledge. This also suggests that the concept "creative problem-solving when an impasse occurs" is itself a threshold concept. The ability of metaphor to convey nonlinguistic aspects of threshold concepts, and to link dissociated ways of cognition, suggests such factors may be a reason threshold concepts are difficult to acquire. The paper recommends that further investigation of this would be worthwhile.

The qualitative results suggest that the students were deeply engaged in the subject matter and were able to construct individual mental models and perspectives on knowledge. The techniques developed in this paper not only shift the emphasis toward active engagement with the content domain but provide a better understanding of teaching expectations to students by showing them ways of being successful. The content domain is sometimes referred to as the knowledge structure and the approaches to knowledge learning is sometimes referred to as the knower structure. Effective learning depends on the interaction between knowledge structures and knower structures (Maton & Chen, 2017). At times, success depends not so much on knowledge structures, but on knower structures. It relies on the student knowing the tacit 'rules of the game' in which the instructor assumes an 'ideal knower' where certain forms of engagement are considered more appropriate than others. Metaphors help by providing insight into how experts, as knowers, address problems. However, the novelty of the metaphor approach was challenging to some students, and more practice is recommended in using it.

More broadly, although academics face challenges due to competing research demands and limited time and resources, active learning is not a simple plug-and-play mechanism. Attitudes also play an important role in how well learning occurs. Exhibit 1 of (Heywood, 2016) compares extended and restricted professionalism in higher education, where restricted professionality in engineering education includes: Instruction (teaching) is considered less important than research. In contrast, extended professionality in engineering education involves instruction (teaching) considered as important as research. Using metaphors can help bridge the gap between teaching and research by exploiting the deep structure of the content used by researchers.

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